# A risk-adjusted valuation model for non-life insurance companies

Francesco Paolo Natale
Dipartimento di Scienze Economiche e Aziendali
Università Milano-Bicocca
*francesco.natale@unimib.it*

***Abstract***

In this paper we present a model for the evaluation of a non-life insurance company.

This model, differently from the others in use (Copeland 2000, Nakada 1999, CAS 1991, D'Arcy and Gorvett 1998) examines dynamically the risk and return of a non-life insurance company. Such models have been extensively applied in the life insurance (e.g. European Embedded Value) but none is applicable to the non-life business.

Up to now, in the non-life industry, some sophisticated models have been recently proposed by actuaries for the evaluation of both portfolio's risk and the stochastic amount of the technical liabilities (Kunkler 2004, 2006). However those models do not give any insight regarding the valuation of the whole insurer and hence on the profitability of the business in the short/medium and long run.

Financial practitioners basically apply two distinct strands of valuation methodology to detect the profitability (see Hodes et al. for a description, 1996). The first deals with direct evaluation of insurer's assets and liabilities and hence net assets (*balance-sheet based*), while the second concerned with an assessment of the future earnings likely to flow from the present assets and liabilities (*cash-flow based*). Usually the balance-sheet is the starting point but some additional assumptions are required in both approaches. These methods often conclude with a specific measure of return and extrapolate some 'basic' measure of risk given by the regressed CAPM beta. However the cash-flow based valuation methods tend to be more flexible to the several constraints imposed by domestic insurance supervisors (e.g. solvency capital) and stakeholders (e.g. minimum dividend yield).

These approaches do not give the correct importance to the risk profile and do not distinguish between insurance companies with similar balance-sheet data but different risk profiles. We propose a stochastic model, similar to the model already applied in the life industry, able to exhibit the risk profile of the insurance company and its profitability over the time. This model basically decomposes the final value in three main components: 1) Net Asset Value in the *Winding-Up* assumption; 2) Value of the Portfolio with the assumption of *Run-off*; 3) Value of the New Business (*Going-Concern* assumption).

The first component is derived adjusting the balance-sheet assets, while the second relies on a stochastic valuation of the cash outflow embedded in the specific type of business, after a risk-free discounting (see England and Verrall 2002 and Mikosch 2006). This is made up using internal data (*loss triangle*) and extrapolating the cash outflows over the time. Notice that the economic valuation of the technical provisions is actually an issue for both life and non-life insurance companies, due mainly to the Solvency II project and to the research of an appropriate accounting standard for the insurance liabilities (so-called Phase 2 based on IFRS 4). Notice also that we are mainly interested in outflow since the biggest part of non-life insurance contracts are annual, therefore we are not required to plan different inflows for the same contract over the time. The third component relies on

realistic and prudent assumption related to the new sales in the future and to the profitability of the new contracts.

### *Introduction*

The application of the recent developments in the risk-adjusted evaluation to the non-life insurance firm is actually an issue for both financial analysts and practitioners. However several steps ahead have been done in the life insurance industry while the non-life industry is still in search of applicable models able to unambiguously evaluate together risk and return. In this paper we efficiently extend some key-findings on the life insurance to the non-life insurance companies.

The literature deals with the issue of the risk-adjusted evaluation in two ways. The first involves the *balance-sheet based methods* based on the application of multipliers to some relevant values of the balance-sheet. Usually the ratio to be investigated is V/P where V is the value of the insurer and P is the gross premium income. These models rely on the decomposition of the V/P ratio into relevant driving factors (eg. underwriting profits and investment profits). These methods distinguish between different insurance companies applying a specific multiplier for each specific business line. Some examples are given in Copeland (2000), SCOR (1991), Sturgis (1981).

The second set of methods (*cash-flow based methods*) greatly relies on the seminal paper of Babbel *et al.* (1997). They identify the main sources of risk for a non-life insurance companies and propose some criteria to be followed for a viable economic valuation model based on cash flow of an insurance company. Furthermore they establish a matrix with four potential models in accordance with the uncertainty to be included in the evaluation. More recently Bingham (2000) discusses some issues about the application of CAPM for the evaluation of cash flow of a non-life insurance companies.

Recently some developments based on the option-pricing theory (OPT) have been applied to the evaluation (see Danhel and Sosik, 2004 and Damodaran, 2002). These models assume that traditional DCF models underestimate the value of real options. Thus, option pricing theory has become a necessary tool to reflect these specific cases in the valuation.

OPT is also recognized in the value of a company as a call option. Stockholders act as holders of an option on the company's assets with a strike price at the level of the company's liabilities.

However some authors (Sommer *et al.*, 1996) argue on the non applicability of OPT because these models correctly reflect the variability of future cash flows, but they require many valuation assumptions, including variability parameters. Furthermore the OPT models are very sensitive to inputs and can be easily manipulated and misused (Hayne, 1999).

### *The proposed model*

Non-life insurance industry is characterized by the risky claims process subject basically to three sources of risks:

1) the occurrence of insurance events;
2) the amounts to be paid;
3) the date of the payment.

The consideration of these three random variables is the main concern for an insurance company due to the assessment process of the technical liabilities. Technically the insurance liabilities represent a prudent estimation of the future payments $\tilde{C}_{ij}$ where *i* is the accident period and *j* is the payment or development year. These outstanding liabilities must include the *best estimate* of the future payments but also a sufficient prudential margin (*risk margin*) to take into account the extreme randomness of the claim process.

Actuaries have proposed several methods for the ‘point-estimation’ of the provisions for future payments. These methods have also been adopted by the accountants in order to establish a prudent and reliable value for the liabilities of the insurance company. However, under the pressure of both Solvency II and IFRS projects, there is more attention on the valuation of technical provisions with financial methods. At least in theory those methods should be based on the cash flows, evaluating and projecting them, but also taking into account a proper buffer for the risky business and a financial discount on the term structure. Notice that the actuarial methods are mainly based on the traditional run-off analysis, providing the point-estimate of the ultimate loss reserve. Examples of these models are the famous chain-ladder method and the Bornhuetter-Ferguson model (see Swiss Re 2000 and Calandro 2004 for an user-friendly introduction) which belongs to the Bayesian family. However the balance-sheet value of these liabilities is typically higher than a fair value for the application of the accounting policies based on prudence.

This distorted valuation of the liabilities makes the balance-sheet methods ‘distorted’, unless financial practitioners can adjust these values dropping accounting policies.

In our model we recall a method based on the financial valuation of these liabilities in order to provide a stochastically calculated *fair value* of these provisions. Therefore our model is fully stochastic and requires the current term structure as the only market information. The first paper on this topic is due to England and Verrall (2002). A similar application can be found in De Felice and Moriconi (2003) and more recently in ISVAP (2006) for the valuation of the fair value of the motor third party liability.

However these authors give only some insights on these methods and apply them to a mono-line insurance company with the assumption of independent payments. They do not extend to the case of correlated payments over the time for the problematic consideration of the dependences in the risk factors affecting the pattern of the future cash outflows.

Such dependencies can be explained with two reasons:

a) *exongenally stated*;

b) *endogenally stated*.

Examples of the first category are the decisions of the courts of justice affecting commercial liabilities and compulsory motor liabilities, while examples of the second are the decisions taken on asset-liabilities considerations looking at movements of interest rates, or decisions taken considering fiscal policies.

We propose the projection of the cash outflow embedded in the existing portfolio of contracts and the discounting of the these cash flows in order to have an estimation consistent with the financial market. However this valuation does not include the sources of risk typical of the claim process over the time.

To include the sources of risk outlined above we use the bootstrapping algorithm proposed by England and Verrall (2002) and recently applied by ISVAP (2006) for the stochastic valuation of the motor liabilities.

Following the approach of EIOPC (2006)[1] we consider as the fair valuation of the technical provisions the $75^{th}$ percentile of the full distribution resulting from the bootstrapping procedure on the cash outflows derived by the run-off triangles. The bootstrapping allows for a non-parametric estimation of the probability distribution of the liabilities, therefore parametric assumptions are not required.

This probability distribution is useful for several purposes. First, for the calculation of an appropriate risk margin to be added to the central (best) estimate for considering the risky nature of the business. Second, obviously, for the calculation of the risk capital to be set aside for the *reserve*

1 The *Draft Amended Framework for Consultation on Solvency* II of the EIOPC (2006) defines the fair valuation of the technical liabilities as the sum of best estimate and risk margin. For the risk margin the EIOPC states: “The risk margin covers the risks linked to the future liability cash flows over their whole time horizon. Two possible ways to calculate the risk margin should be considered as working hypotheses. It can be calculated as the difference between the 75th percentile of the underlying probability distribution until run-off and the best estimate. Alternatively, the risk margin can be calculated based on the cost of providing SCR capital to support the business-in-force until run-off. Further quantitative impact information should be collected to assess the merits of the two methods.”

*risk*, usually called *reserve capital*. This amount of capital is aimed to absorb unforeseen losses over an identified $\alpha$% confidence level in a given time period (typically 1 year).

The positive (negative) difference with the statutory liability value (it is likely it is higher than the sum of best estimate and risk margin) embodies a stream of future profits (losses) in the existing portfolio of contracts. We call this VEC (*Value of Existing Contracts*). This value can be efficiently thought as the analogous of the VIF (*Value – In-Force*) in the life industry.

The sum of the NAV (Net Asset Value) adjusted with market value of assets (*Winding Up* assumption) and VEC provides an evaluation at *t* of the future cash inflows and outflows. Notice that these cash flows refer to the existing portfolio and therefore they tend to zero when these contracts expire and all the cash flows related to these contracts have been paid (*Run-Off assumption*). For simplicity we call this value VoRO (*Value of Run-Off*).

However it is widely known (see Kaufmann, 2001 and Alghrim, 2005) that an important fraction of these contracts are renewed form *t+1* on and also that the sales forces act to expand their customers' base.

This growth effect produces at least two kind of risks:

1) the risk that the collected premiums in the year *t+1* are insufficient to cover the paid losses at the end of the year *t+1* for the new business;
2) the risk of underestimate, at the end of year *t+1* (date for setting up the liabilities coming from the contracts concluded in the year *t+1*), the claims to be paid in the year *t+2, t+3, t+k* referred to the new business collected in the accounting year *t+1*.

These risks are together considered as *premium risk*, even though the second part can be thought as the *reserve risk* for the new business.

The earned premiums in *t+1* should be at least sufficient to cover the paid losses in *t+1* and the losses to be paid in the next years (*t+2, t+3, t+k*) but properly valuated in *t+1* and stated in the technical liabilities. Therefore the earned premiums in *t+1* can produce a loss or a profit at the end of the generation *t+1* of contracts. Notice that the valuation of the proper liability to set aside for the *t+1* generation requires again the bootstrapping procedure for estimated claims to be paid in year *t+2, t+3* and referred to the generation *t+1*.

The repetition several times of this last procedure for the earned premiums in *t+2, t+3, t+k* provides a consistent estimation of the Value of New Business (*VNB*). It is worth emphasizing that this last component is greatly uncertain because it depends upon several crucial assumptions, i.e. growth rates, loss ratios for the next generations, fraction of claims not paid each future year.

To summarize we propose a model that decompose the value of a non-life insurance company in three parts:

1) Net Asset Value (*NAV*) assuming the *Winding-Up* situation;
2) Value of Existing Contracts or *VEC* assuming the *Run-Off* situation;

The sum of 1) and 2) is the *Value of Run-Off* (*VoRO*).

3) Value of New Business (VNB).

A key feature of our model is the bootstrapping procedure. We follow the method proposed in England and Verrall (2002) and recently recalled in ISVAP (2006), QIS – Technical Notes - (2006).

This method try to stress the run-off triangle in order to derive the cash outflow projection for the existing contracts. These projections are derived using generalized linear model estimates (GLM). GLM is used since each log of future payment is decomposed in two components:

a) *generation's component (i);*
b) *year's payment component (j);*

In formula the assumption under this model is that each incremental claim by year is distributed as an Over-Dispersed Poisson of parameter $\phi$ (parameter of overdispersion) with mean and variance given by:

$$E[C_{ij}] = \hat{M}_{ij} = x_i \cdot y_j$$
$$Var[C_{ij}] = \phi \cdot x_i \cdot y_j \qquad (1)$$

where $x_i$ is the total amount of paid claims for each year of payment ($i$) and $y_j$ is the fraction of the total amount of claims paid each year. The over-dispersion parameter ($\phi$) is introduced to overcome the restrictive Poisson's assumption of the same mean and variance.

Relaxing this statistical restriction allows for a greater level of elasticity and a better fit to real data. We statistically estimate these parameters under the straightforward constraint that $\sum_{k=1}^{n} y_k = 1$.

It is also worth noting that this model allows for some negative payments even though the total payment (cash outflow) per year must be positive. Using the introduced symbols, $x_i$ must be positive while $y_j$ can be positive or negative[2]. Thus this model cannot be applied if some $x_i$ are negative. A negative $x_i$ produces a negative variance, computationally intractable[3].

To estimate these parameters the model is usually reparametrized to achieve a linear expression of (1). The logical function for the reparametrization is the log. The log allows for rearranging (1) in (2) and thus estimating a linearized function.

$$log(\hat{M}_{ij}) = c + \alpha_i + \beta_j \qquad (2)$$

The estimation of the parameters is done using the Generalized Linear Model (GLM) routine with *quasi-likelihood* in order to permit negatives and non rounded figures.

The model is applied to the loss triangle of historical cash outflows made up by $n$ years of generation and $n$ years of payments. Therefore we have $2n-1$ parameters to estimate.

For the bootstrapping we use the Pearson residuals to model the underlying distribution as in McCullagh and Nelder (1989). Dropping the suffices indicating the origin year $i$ and development year $j$, the Pearson residuals $r_P$ are defined as:

$$r_P = \frac{C - \hat{M}}{\sqrt{\hat{M}}} \qquad (3)$$

where $\hat{M}$ is the fitted incremental claims given by equation (1). The vector of the Pearson residuals is composed by $n(n+1)/2$ observations. Next these Pearson residuals between the theoretical underlying model (1) and the real data $C_{ij}$ have to be adjusted for the number of parameters to be estimated and the number of available observations[4].

The bootstrap process involves resampling with replacement from the residuals (Efron and Tibshirani, 1993). A bootstrap data sample ($C^*$) is easily created by inverting equation (3), using the resampled residuals ($r_P^*$), together with the fitted values. Given a resampled Pearson residual $r_P^*$,

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[2] Negative payments from the liabilities can be due to the reinsurance if the net reserves are considered or, in some business lines, to the recoveries.

[3] With this limit the model is not applicable if the reinsurance payments are higher than the insurer's payments, or in these lines of business in which the recoveries can be numerically higher than the payments.

[4] The adjustment is a rescaling of these residuals thought the over-dispersion parameter $\phi$. Technically $\phi$ is the sum of the Pearson residuals squared divided by the degrees of freedom where the degrees of freedom is the number of observations minus the number of parameters estimated.

together with the fitted value $\hat{M}$, the associated bootstrap incremental claims amount $C^*$ is given by:

$$C^* = r_P^* \cdot \sqrt{\hat{M}^*} + \hat{M}^* \qquad (4)$$

Having obtained the bootstrap data sample ($C^*$), the model is refitted and the statistics of interest calculated. The replication of the process variance is achieved by simulating an observed claims payment for each future cell in the left-bottom run-off triangle. The simulation is run with the bootstrap value as the mean and under the assumption of the process distribution assumed in the underlying model which in this case is over-dispersed Poisson.

The algorithm to obtain random numbers from an over-dispersed Poisson with mean and variance given by (1) and over-dispersion parameter $\phi$ is due to the recent contribution of Madsen and Dalthorp (2005). These authors propose a method to generate under and over-dispersed random variables with a given variance-covariance matrix. Particularly they use the intuition behind the *Overlapping Sums* (OS), extended to matrix of data by Park e Shin (1998).

We use their algorithm to obtain vector of simulated payments or cash outflow for each 'payment year *j*' for each 'payment year *i*'.

Strongly correlated count, under or over-dispersed random variables can be simulated from the OS algorithm, provided that the random variables have similar means and variances.

Once we calculate the distributions of the technical liabilities for each business lines we can have a comparison with the statutory liability value. The positive (negative) difference represents a stream of future profits (losses) in the existing portfolio of contracts and we call it Value of Existing Contracts (VEC).

The last step requires some assumptions regarding the frequency and the severity of the claim process. Otherwise an assumption on the total cost of claims is required. Several methods for both approaches have been proposed trough the years. A complete reference on these topics can be found in Wang (1998) and Meyers (1999).

For modeling the total cost of claims we use a Monte Carlo simulation based on internal data assuming a lognormal severity distribution and the negative binomial distribution for the number of claims. These distributions are typical in the literature on the loss distributions (see Klugman *et al*. 2004), otherwise other parametric assumptions can be made.

Notice that to each cost of claim process we need to apply again the bootstrapping process in order to have the estimation of the probability distribution of the technical liability and therefore the best estimate and an appropriate risk margin. The difference between the earned premiums and the cost of claims is called Value of New Business (*VNB*) and represents the stream of profits from the new generations of contracts. Several assumptions are required to define the earned premiums and the total cost of claims over the time. These assumptions must be prudent and should be elaborated according to the strategic plan.

In the next session we give an application of our model in order to decompose the value of a non-life insurance company.

***Empirical application***

The non-life insurance company has the following simplified balance-sheet at the end of year *t* and profit and loss account for the year *t+1*.

| | |
|---|---|
| Assets (fixed income) | 125000 |
| Technical liabilities (not discounted) | 95000 |
| Net Asset value | 30000 |

| | |
|---|---|
| Premium Income | 50000 |
| Cost of Claims | 30000 |
| of which | (25000) for technical liabilities |
| | ( 5000) paid |
| Operating expenses | 15000 |
| Profit | 5000 |

The underlying assumptions are on the structure of the business lines. Technical liabilities come from one line of business. We limit the valuation of the *VNB* to the first year ($t+1$). However including basic assumptions on the growth rates of premiums, cost of claims and expenses the model can be easily extended to future years . Table 1 reports the run-off triangle highlighting the $C_{ij}$ .

Table 1: Line of Business 1 (LoB1)

| *Payment year* | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| *Accident year* | | | | | | | | | | |
| ***1996*** | 5012 | 3257 | 2638 | 898 | 1734 | 2642 | 1828 | 599 | 54 | 172 |
| ***1997*** | 106 | 4179 | 1111 | 5270 | 3116 | 1817 | -103 | 673 | 535 | |
| ***1998*** | 3410 | 5582 | 4881 | 2268 | 2594 | 3479 | 649 | 603 | | |
| ***1999*** | 5655 | 5900 | 4211 | 5500 | 2159 | 2658 | 984 | | | |
| ***2000*** | 1092 | 8473 | 6271 | 6333 | 3786 | 225 | | | | |
| ***2001*** | 1513 | 4932 | 5257 | 1233 | 2917 | | | | | |
| ***2002*** | 557 | 3463 | 6926 | 1368 | | | | | | |
| ***2003*** | 1351 | 5596 | 6165 | | | | | | | |
| ***2004*** | 3133 | 2262 | | | | | | | | |
| ***2005*** | 2063 | | | | | | | | | |
| Total | 23892 | 43644 | 37460 | 22870 | 16306 | 10821 | 3358 | 1875 | 589 | 172 |

From this run-off triangle the parameters $\alpha$s, $\beta$s, the constant $c$ and the over-dispersion parameter $\phi$ have been estimated trough GLM routine.

Table 2 reports the GLM parameter estimates of equation (2) for the insurance company.

Table 2: GLM estimates of the parameters for the Over-dispersed Poisson model

| constant $c$ | 8.46653 | | $\phi$ | 195.06 | | | | |
|---|---|---|---|---|---|---|---|---|
| $\alpha_2$ | $\alpha_3$ | $\alpha_4$ | $\alpha_5$ | $\alpha_6$ | $\alpha_7$ | $\alpha_8$ | $\alpha_9$ | $\alpha_{10}$ |
| 0.0480 | -0.0331 | 3.51145E-05 | 0.1198 | 0.0720 | 0.2041 | 0.2202 | 0.2696 | 0.0334 |
| $\beta_2$ | $\beta_3$ | $\beta_4$ | $\beta_5$ | $\beta_6$ | $\beta_7$ | $\beta_8$ | $\beta_9$ | $\beta_{10}$ |
| 0.1529 | 0.3121 | -0.3343 | -0.9167 | -1.3181 | -1.4760 | -1.3910 | -1.6248 | -1.5885 |

With these parameters the bootstrapping procedure has been applied in order in order to obtain the full stochastic distribution of the liabilities of this portfolio of contracts. The iterative simulation procedure of the resampling with replacement has been performed 10000 times.

Alternatively specific assumption of the liability distribution is required in order to analytically derive the accepted fair valuation[5]. In this context the fair valuation is the 75% percentile of the full distribution. The 75$^{th}$ percentile is deemed to include a reliable prudence margin over the best estimate (the mean) of the future liabilities.

From the estimation of the payments to be made in the future ($C^*$) on the existing portfolio an estimate of the cash outflow from year $t+1$ until year $t+k$ is derived. The sum of the payments on the right-bottom diagonals[6] of the triangle in Table 1 represents the cash outflow in the future years

[5] This is the parametric approach introduced by Mack (1994). The usual assumption is the lognormal distribution of the future payments with parameters calibrating on the data. Given these parameters it is easy to calculate the desired (usually 75$^{th}$) percentile.

[6] The right-bottom part of the triangle in Table 1 has been completed using the Chain-Ladder method illustrated in Calandro (2004) and widely applied by actuaries for the valuation of technical liabilities.

deriving from the actual portfolio. In formula it is straightforward to see that the future cash flows for year $t+1$, $t+2$, $t+3,\dots,t+k$ are:

$$CF^{*}_{t+l} = \sum_{i=1}^{k-l} C^{*}_{k+1-i,i+l} \quad (l = 1,\dots k\text{-}1) \tag{4}$$

In our example the application of (4) gives the stochastic distribution of the nine future cash flows. These cash flows have to be discounted, according to the classical financial valuation principles, using the term structure of the interest rates.

To have a proper discount of the future cash flows we use the model based on the extraction of the main principal components from the entire dataset of the historical variation of the interest rates. This model has been proposed by Littermann and Sheinkman (1991) and, among others, recently tested on the European interest rates by Braga (2006). Principal component analysis is a technique for multivariate analysis used to investigate volatility in a set of correlated variables (i.e. interest rates). Its application enables us to re-express the original data set of monthly changes in spot rates with different maturities in a number of linear combinations of the original variables. Those variables, namely the principal components, are mutually orthogonal and receive a specific meaning. The first principal component explains the shift of the term structure. The second represents the changes in the slope while the third principal component refers to the curvature changes[7].

We run a Monte Carlo simulation on the first three principal components in order to obtain a set of term structure to be used for discount purposes. This simulation is run in order to produce 10000 different term structures drawing multivariate normally distributed random variables for the shocks in each principal component. Obviously the orthogonality of the components requires a zero covariance matrix for the simulation. Notice that more complicated models can be adopted to include the effect of discounting with stochastic interest rates. Examples are in Kaufmann et al. (2001) and Alghrim (2004). However these authors highlight that complex interest models are particularly useful for the pricing of interest rate derivatives but can be redundant for a set of future cash flows.

Discounting each vector of 10000 cash flows for each maturity $l$ ($l$=1,….$k$-1) we obtain the discounted cash outflow for each future year originated by the existing portfolio. The sum of these cash outflows gives the discounted *fair value* of the liabilities. The positive (negative) difference with the statutory liability value (95000) sums up a stream of future profits (losses) in the existing portfolio of contracts. This is called the VEC (*Value of Existing Contracts*).

Table 3 reports the descriptive statistics for the distribution of both undiscounted and discounted future payments. Exhibit 1 shows the undiscounted stochastic distribution of the liability using the OdP-bootstrapping method and its cumulative distribution function for the calculation of the 75$^{th}$ percentile while Exhibit 2 reports the distribution of the nine cash outflows (undiscounted) implicit in the OdP-bootstrapping method.

[7] Further details on the choice of principal components and on the original data are available in the Annex A.

Table 3: Descriptive Statistics for discounted and undiscounted future payments

| *Statistics* | *Undiscounted* | *Discounted* |
|---|---|---|
| mean | 85621 | 82399 |
| median | 84857 | 81710 |
| std deviation | 11616 | 11185 |
| skewness | 0.3125 | 0.312 |
| kurtosis | 3.1078 | 3.116 |
| 60th percentile | 87988 | 84591 |
| 70th percentile | 91423 | 87994 |
| 75th percentile | 93135 | 89613 |
| 80th percentile | 95212 | 91625 |
| 90th percentile | 100730 | 96928 |
| 95th percentile | 105790 | 101770 |

Exhibit 1: Stochastic distribution of the liabilities

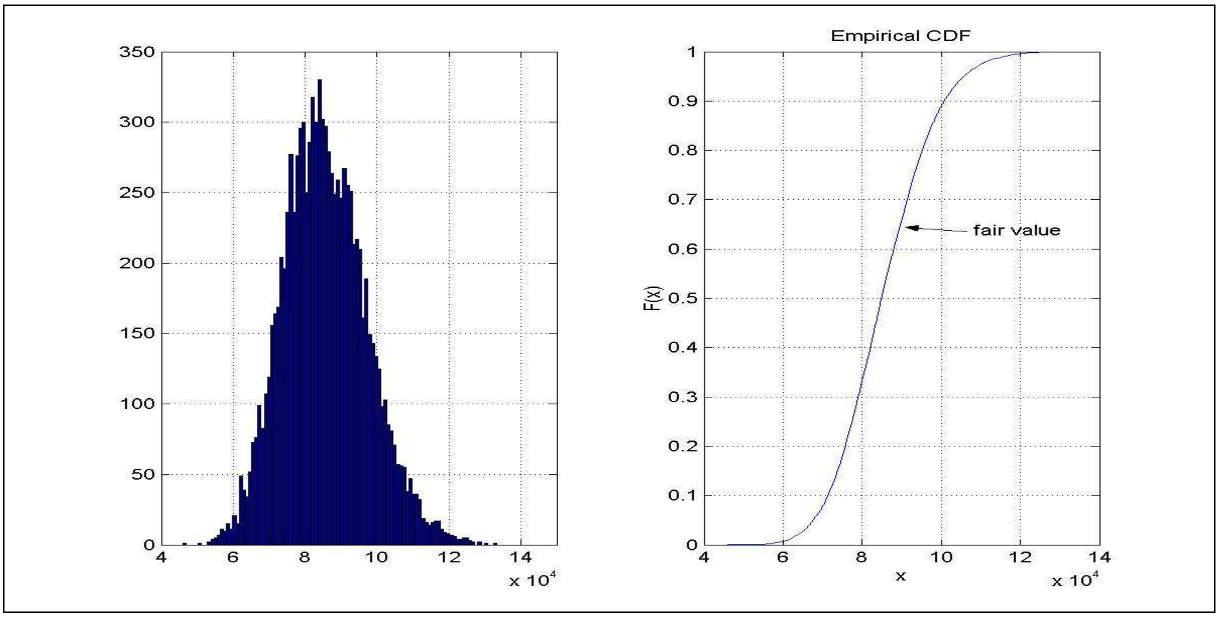
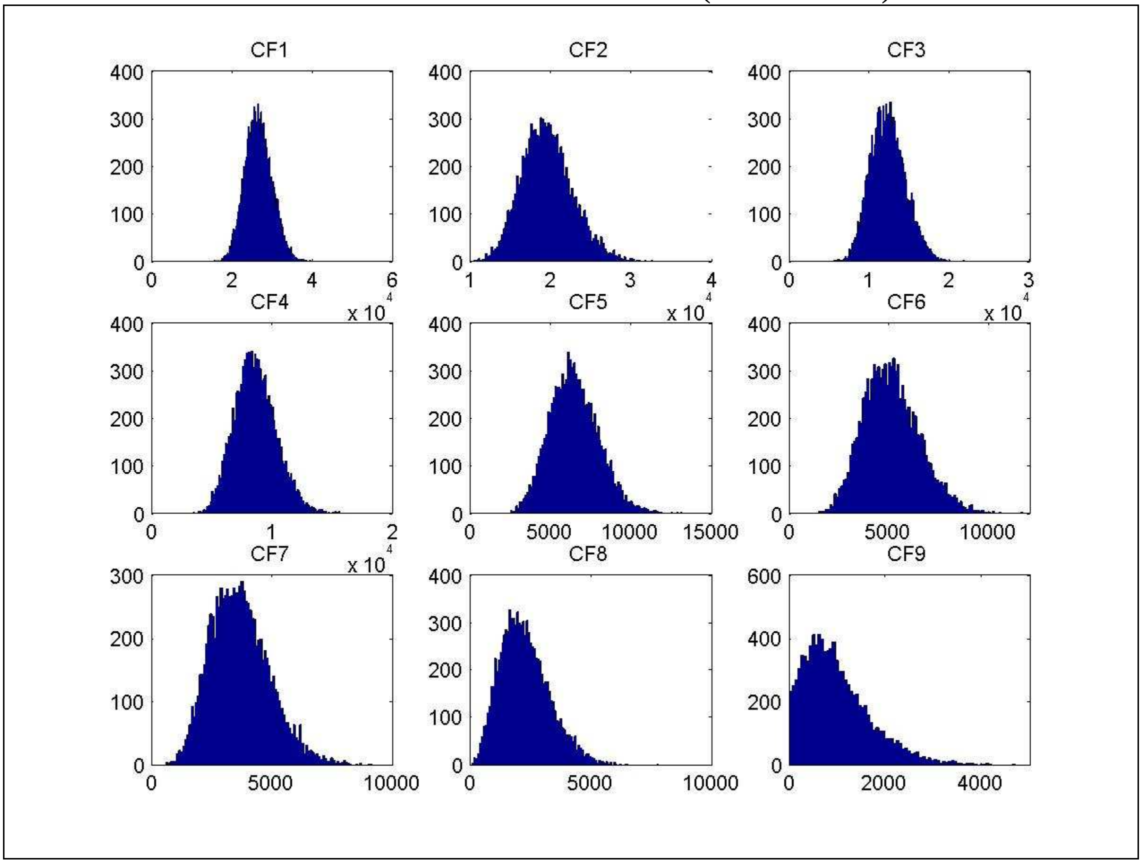


Exhibit 2: Stochastic distribution of the cash outflows (undiscounted)




The last step requires some basic assumptions on the distribution of the cost of claim (severity) and on the number of adverse events (frequency). Following the growing literature on the insurance risk and on operational risk we assume that the cost of claim is lognormally distributed with coefficient of variation (CV) equal to 3. The assumptions are made up only for operational purposes. However they can be easily removed and adapted to the historical experience of the insurance company.

We assume that the total cost of claims (in mean) is given by 5000 events (number of claims) and each claim costs on average 6. CV is greatly important in the insurance practice in order to define the exposure of each contract and the reinsurance limits in the non-proportional treaties. With this information we can easily calculate the standard deviation of the severity distribution and, solving the non-linear system for the lognormal parameters, also the estimates of the parameters for the lognormal distribution.

If $\tilde{S}$ is the random variable representing the cost of each claim we can derive the parameters $\mu_S$ and $\sigma_S$ ($\mu_S$ = 0.6404 and $\sigma_S$ = 1.5174).

$$\tilde{S} = \sum_{i=1}^{\tilde{n}} \tilde{X}_i \qquad (5)$$

where $\tilde{X}$ is the random cost of claim and $\tilde{n}$ is the random number of claims.

The number of claim (frequency) is a Negative Binomial distributed random variable with mean 5000 and variance (5000/2)^2=6250000. We used these standard assumptions in the insurance literature but, again, different parameters or distributions can be easily included. The Negative Binomial distribution is more flexible than the Poisson distribution since the latter has the known limit of equality between mean and variance.

The iterative procedure involves a Monte Carlo simulation to derive the total cost of claims and then the ODP-bootstrapping procedure on the fraction of the cost of claim not paid in *t+1* (25000/30000). However we cannot consider the value of the new business (VNB) as the profit since we are not including the cash flow randomness and the fair valuation on the term structure.

For simplicity we assume that the pattern of the future payments for the new generation of the contracts is unchanged. In other words the run-off triangle showed in Table 1 remains the same even though we have to rescale it in order to introduce the new accident year (2006) and drop the oldest (1996).

The second application of the OdP-bootstrapping to this new triangle enables us to obtain the cash outflow (undiscounted) projections for the years *t+2, t+3, …., t+k+1* from the sum of new and old contracts but evaluated one year later. Recall that the cash outflow for the new contracts in *t+1* is 5000 and it is known.

However we are interested in the calculation of the cash outflow only related to the new business. It can be easily obtained by differencing the cash outflow of the new contracts (underwritten in *t+1*) calculated in the next year (*t+2*) and the cash outflow of the existing contracts in the projection year (*t+2*). For example the cash outflow of the new contracts in *t+2* is made up by the difference between the cash outflow of the entire portfolio (existing plus new contracts) calculated in t+2 and the only cash flow of the existing contracts in *t+2*.

For the sake of brevity we drop the graphs related to the distributions of the future cash outflows from t+1 on including the new generation of contracts. In table 4 we report the descriptive statistics of the cash outflow with and without the new generation of contracts and, by difference, the descriptive statistics of the cash outflow of the new business contracts.

Table 4: descriptive statistics of the cash outflows

| | *t+1* | *t+2* | *t+3* | *t+4* | *t+5* | *t+6* | *t+7* | *t+8* | *t+9* | *t+10* |
|---|---|---|---|---|---|---|---|---|---|---|
| | ***Cash Flow 1*** | ***Cash Flow 2*** | ***Cash Flow 3*** | ***Cash Flow 4*** | ***Cash Flow 5*** | ***Cash Flow 6*** | ***Cash Flow 7*** | ***Cash Flow 8*** | ***Cash Flow 9*** | ***Cash Flow 10*** |
| ***Existing contracts*** | | | | | | | | | | |
| *Mean* | 26597 | 19576 | 12295 | 8608 | 6467 | 5091 | 3682 | 2282 | 1021 | |
| *Std deviation* | 3387 | 3184 | 2187 | 1744 | 1528 | 1395 | 1240 | 1028 | 734 | |
| *Skewness* | 0.2256 | 0.3724 | 0.3269 | 0.3704 | 0.3983 | 0.4528 | 0.5614 | 0.6780 | 1.2909 | |
| *Kurtosis* | 3.0612 | 3.2187 | 3.1681 | 3.3056 | 3.2925 | 3.2332 | 3.4347 | 3.5156 | 5.4986 | |
| ***Existing + New contracts*** | | | | | | | | | | |
| *Mean* | | 25439 | 19175 | 12154 | 8314 | 6341 | 4733 | 3430 | 1848 | 1050 |
| *Std deviation* | | 2797 | 2582 | 1880 | 1552 | 1375 | 1227 | 1085 | 802 | 661 |
| *Skewness* | | 0.1875 | 0.3086 | 0.2682 | 0.3347 | 0.4136 | 0.4472 | 0.5638 | 0.7102 | 1.0406 |
| *Kurtosis* | | 3.1325 | 3.1242 | 3.1633 | 3.1072 | 3.2861 | 3.2217 | 3.4201 | 3.7221 | 4.5553 |
| | ***CF NEW BUSINESS*** | | | | | | | | | |
| ***New contracts*** | | | | | | | | | | |
| *Mean* | 5000 | 5863 | 6881 | 3546 | 1846 | 1250 | 1051 | 1148 | 827 | 1050 |
| *Std deviation* | | 4219 | 3410 | 2579 | 2190 | 1955 | 1736 | 1500 | 1090 | 661 |
| *Skewness* | | (0.1328) | 0.0377 | (0.0279) | (0.0199) | (0.0385) | (0.0546) | (0.0154) | (0.1434) | 1.0406 |
| *Kurtosis* | | 3.0087 | 3.1021 | 3.0430 | 3.0843 | 3.1546 | 3.1730 | 3.2643 | 3.6752 | 4.5553 |

Notice also that the VNB can be easily obtained by differencing the premium income in *t+1* and the discounted cash outflow of the new business contracts. Cash flows from the new business have been discounted using the discounting rates generated with the principal components simulation explained above. With this method the entire distribution of the VNB is easily achievable. This distribution is unambiguously related to the historical patterns of payments and to the selection and selling policies (eg distribution channels) of the insurance company. Changes on these relevant variables imply different VNBs. Computationally the undiscounted VNB is derived as the difference between the cash outflows of the old and new contracts (undiscounted) and the cash outflows of the old contracts (undiscounted). This implies the consideration of these new contracts in a on-going business model (going-concern assumption). An alternative approach could be to evaluate this new stream of cash outflows separately from the old ones already scheduled. However this second method requires considerably more assumptions than the model here illustrated and thus additional sources of uncertainty have to be considered. The statistical distribution of VNB in this example has 12421 as mean and a standard deviation of 12594, positive skewness of 0,0697 kurtosis 3.3684.

Table 5 summarizes the decomposition of the value of this non-life insurance company in its fundamental components. For the sake of simplicity the Net Asset Value (*NAV*) has been kept equal to the book value. Changes can be easily included re-evaluating properly the assets in order to consider their fair value.

Table 5: decomposition of the risk-adjusted value

| | |
|---|---|
| ***1) Net Asset Value*** | **30000** |
| 2) Technical Liabilities (not discounted) | 95000 |
| 3) Fair Value Technical Liabilities (discounted) | 89613 |
| ***4) Value of Existing Contracts (VEC) (3 – 2)*** | **5387** |
| ***5) Value of Run-Off (VoRO) (4 + 1)*** | **35387** |
| ***6) Value of New Business (VNB)*** | **12421** |
| | |
| ***Risk-Adjusted Value*** | **47808** |

## Conclusion

In this paper we present a model to analytically derive the risk-adjusted value of a non-life insurance company. We restrict our analysis to a mono-business insurance company, however several business units can be easily included by aggregating the streams of cash outflows from the existing portfolios.

The model here presented separately evaluates the Net Asset Value considering the fair valuation of its assets (mainly bonds and real-estate), the value of the existing contracts (VEC) and the value of the new business (VNB). The first component requires an analysis of the portfolio of the existing assets in order to replace the book value with their market (fair) value.

The second component greatly relies on the determination of the future cash outflow from the existing portfolio of contracts. For this purpose we use the overdispersion (OdP) Poisson algorithm with the bootstrapping. An useful OdP algorithm has been recently presented by Madsen and Dalthorp (2005). Their algorithm interestingly allows also for correlations between the over-dispersed random variables. This additional feature can be useful for stress testing or for specific business lines with high level of dependence. The bootstrapping algorithm allows to derive the complete distribution of the future cash outflow then used to calculate an undiscounted fair value. Notice that this is a non-parametric method and *a priori* assumptions on the distribution of these cash flows are not required. These cash flows allow have been then explicitly discounted and summed up in order to calculate the fair value of the liabilities. The positive (negative) difference between the balance-sheet value of the technical liabilities and its correspondent fair value represents a potential stream of profits (losses) from the existing portfolio.

The same procedure can been applied to an additional new generation of contracts to be assumed in the future years. Through these assumptions we can easily insert the new generation of contracts in the existing portfolio in order to calculate a new fair value of the liabilities based on the existing portfolio and based on new contracts. We also provide an example of this method.

It is worth emphasizing that this method, with additional information on the asset side, can be efficiently used for asset-liability management purposes and for capital management aims. For example we could be interested in checking the matching of the scheduled cash inflow from the asset-side with the stochastic cash outflow from the liability-side in order to derive explicit risk measure for the surplus. We could also test alternative investment policies checking the associated risk-return contents.

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## Annex A: interest rates and principal component analysis

Table A.1: Correlation matrix for interest rates with different maturities

| | 1y | 2y | 3y | 4y | 5y | 6y | 7y | 8y | 9y | 10y |
|---|---|---|---|---|---|---|---|---|---|---|
| 1y | 1.0000 | 0.9650 | 0.8480 | 0.7848 | 0.7397 | 0.7116 | 0.6741 | 0.6365 | 0.6009 | 0.5441 |
| 2y | 0.9650 | 1.0000 | 0.9407 | 0.8904 | 0.8520 | 0.8241 | 0.7815 | 0.7549 | 0.7206 | 0.6650 |
| 3y | 0.8480 | 0.9407 | 1.0000 | 0.9888 | 0.9693 | 0.9497 | 0.9149 | 0.8984 | 0.8697 | 0.8200 |
| 4y | 0.7848 | 0.8904 | 0.9888 | 1.0000 | 0.9941 | 0.9822 | 0.9536 | 0.9440 | 0.9196 | 0.8762 |
| 5y | 0.7397 | 0.8520 | 0.9693 | 0.9941 | 1.0000 | 0.9957 | 0.9748 | 0.9710 | 0.9517 | 0.9160 |
| 6y | 0.7116 | 0.8241 | 0.9497 | 0.9822 | 0.9957 | 1.0000 | 0.9841 | 0.9853 | 0.9707 | 0.9416 |
| 7y | 0.6741 | 0.7815 | 0.9149 | 0.9536 | 0.9748 | 0.9841 | 1.0000 | 0.9845 | 0.9765 | 0.9595 |
| 8y | 0.6365 | 0.7549 | 0.8984 | 0.9440 | 0.9710 | 0.9853 | 0.9845 | 1.0000 | 0.9970 | 0.9828 |
| 9y | 0.6009 | 0.7206 | 0.8697 | 0.9196 | 0.9517 | 0.9707 | 0.9765 | 0.9970 | 1.0000 | 0.9933 |
| 10y | 0.5441 | 0.6650 | 0.8200 | 0.8762 | 0.9160 | 0.9416 | 0.9595 | 0.9828 | 0.9933 | 1.0000 |

Exhibit A.2: Percentage of Explained Variance by Principal Components

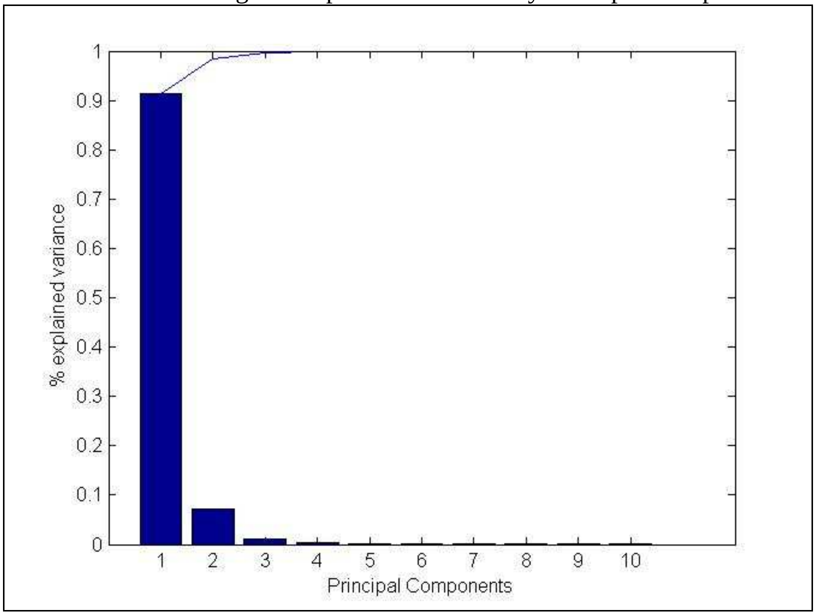
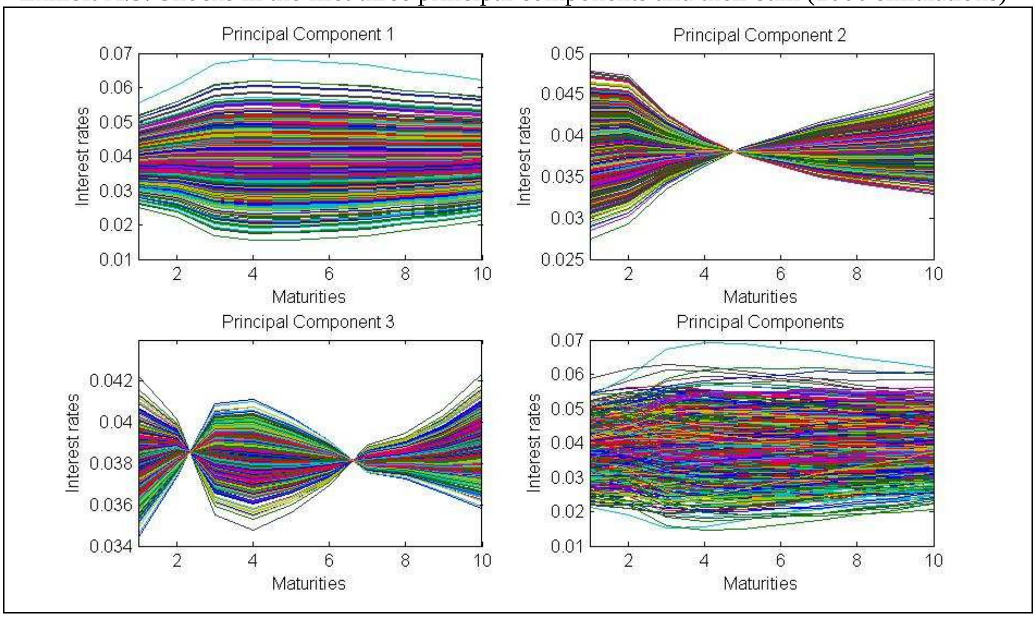



Exhibit A.3: Shocks in the first three principal components and their sum (1000 simulations)



# Annex B: cash flow analysis

Table B.1: Estimate of the technical liability for existing contracts (Chain-Ladder method)

| Payment year | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | Technical Liability |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Accident year | | | | | | | | | | | |
| 1996 | 5,112 | 11,068 | 16,790 | 19,690 | 22,223 | 23,428 | 24,319 | 25,318 | 26,565 | 27,535 | - |
| 1997 | 5,030 | 10,659 | 17,000 | 21,905 | 24,272 | 24,858 | 25,398 | 27,201 | 27,873 | 28,891 | 1,018.49 |
| 1998 | 4,150 | 8,462 | 14,693 | 18,217 | 19,943 | 22,042 | 24,027 | 24,793 | 25,699 | 26,638 | 1,844.59 |
| 1999 | 4,437 | 10,269 | 16,358 | 19,678 | 22,570 | 23,500 | 24,447 | 25,629 | 26,565 | 27,536 | 3,089.47 |
| 2000 | 6,331 | 12,234 | 19,405 | 23,397 | 24,610 | 26,333 | 27,558 | 28,891 | 29,946 | 31,040 | 4,707.15 |
| 2001 | 5,659 | 12,046 | 19,153 | 22,654 | 23,739 | 25,106 | 26,273 | 27,544 | 28,550 | 29,594 | 5,854.83 |
| 2002 | 6,298 | 13,407 | 21,569 | 24,759 | 27,090 | 28,650 | 29,982 | 31,432 | 32,580 | 33,771 | 9,011.78 |
| 2003 | 5,135 | 11,302 | 20,921 | 25,162 | 27,530 | 29,116 | 30,470 | 31,944 | 33,110 | 34,320 | 13,398.93 |
| 2004 | 5,381 | 13,476 | 21,979 | 26,434 | 28,922 | 30,588 | 32,010 | 33,559 | 34,785 | 36,056 | 22,580.09 |
| 2005 | 4,914 | 10,641 | 17,356 | 20,874 | 22,839 | 24,154 | 25,277 | 26,500 | 27,468 | 28,471 | 23,556.89 |
| | | | | | | | | | | | 85,062.21 |
| Development factors | | 2.1652 | 1.6310 | 1.2027 | 1.0941 | 1.0576 | 1.0465 | 1.0484 | 1.0365 | 1.0365 | |

Table B.2: Estimate of the technical liability for existing and new contracts (Chain-Ladder method) 1 year later

| Payment year | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | Technical Liability |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Accident year | | | | | | | | | | | |
| 1997 | 5,030 | 10,659 | 17,000 | 21,905 | 24,272 | 24,858 | 25,398 | 27,201 | 27,873 | 28,891 | - |
| 1998 | 4,150 | 8,462 | 14,693 | 18,217 | 19,943 | 22,042 | 24,027 | 24,793 | 25,699 | 26,638 | 939.05 |
| 1999 | 4,437 | 10,269 | 16,358 | 19,678 | 22,570 | 23,500 | 24,447 | 25,629 | 26,407 | 27,372 | 1,742.60 |
| 2000 | 6,331 | 12,234 | 19,405 | 23,397 | 24,610 | 26,333 | 27,558 | 28,957 | 29,835 | 30,926 | 3,368.09 |
| 2001 | 5,659 | 12,046 | 19,153 | 22,654 | 23,739 | 25,106 | 26,325 | 27,662 | 28,501 | 29,542 | 4,436.52 |
| 2002 | 6,298 | 13,407 | 21,569 | 24,759 | 27,090 | 28,667 | 30,059 | 31,585 | 32,544 | 33,733 | 6,643.28 |
| 2003 | 5,135 | 11,302 | 20,921 | 25,162 | 27,399 | 28,995 | 30,403 | 31,947 | 32,916 | 34,119 | 8,956.73 |
| 2004 | 5,381 | 13,476 | 21,979 | 26,520 | 28,878 | 30,560 | 32,044 | 33,671 | 34,692 | 35,960 | 13,980.91 |
| 2005 | 4,914 | 10,641 | 17,502 | 21,118 | 22,996 | 24,335 | 25,517 | 26,812 | 27,626 | 28,635 | 17,994.22 |
| 2006 | 5,000 | 10,826 | 17,807 | 21,486 | 23,396 | 24,759 | 25,961 | 27,279 | 28,107 | 29,134 | 24,133.66 |
| | | | | | | | | | | | 82,195.04 |
| Development factors | | 2.1652 | 1.6448 | 1.2066 | 1.0889 | 1.0582 | 1.0486 | 1.0508 | 1.0303 | 1.0365 | |