

Efficiency and Liquidity in the Italian Index Option Market

Angela Gallo^a

^a*Researcher, Università degli Studi di Salerno, Department of Management, Via Ponte Don Melillo, Fisciano (SA) (Italy), angallo@unisa.it*

Abstract

Il lavoro si inserisce nel filone delle indagini sulla microstruttura dei mercati finanziari e propone, con specifico riferimento al mercato dei future e delle opzioni, una metodologia di verifica quantitativa delle relazioni funzionali fra liquidità del mercato ed efficienza dei prezzi. Sul piano metodologico, l'indagine sviluppa, preliminarmente, la costruzione di un modello che, muovendo dalla relazione di parità put-call di Black e Scholes, delinea un criterio di identificazione delle opportunità di arbitraggio fra i prezzi delle opzioni e quelli del corrispondente mercato future ed adduce alla definizione di un sintetico indicatore numerico delle violazioni dell'efficienza del pricing, riferito al mercato delle opzioni e calibrato sul mercato future e su quello del sottostante. L'indicatore - che viene definito in un contesto metodologico (prezzi di mercato) indipendente dal modello di pricing eventualmente adottabile - è del tutto oggettivo e di facile costruzione, sia per quanto attiene alla fase della sua quantificazione preliminare (nella quale esso viene definito senza tenere conto dei costi di transazione) sia per quanto attiene alla fase del suo adeguamento sulla base dei bid-ask. Dall'analisi congiunta delle dinamiche dell'indicatore di efficienza e di quelle di indicatori del grado di liquidità si traggono significativi riferimenti per la verifica della relazione oggetto di indagine. L'analisi teorica, basata sul predetto impianto, viene applicata ad un campione di dati riferiti al mercato italiano ed in particolare al mercato delle opzioni e dei future sull'indice S&P/MIB con il duplice scopo di testare, da un lato, l'efficienza del pricing dei suddetti mercati e, dall'altro, l'applicabilità sistematica del modello di indagine proposto.

The paper investigates the pricing efficiency of the Italian option index markets in an ante-crisis period. Following the well-known model-free approach based on the put-call parity (PCP) of Black and Scholes, we develop a new cross-market methodology for the definition of the violations/inefficiency of the market (not only referable to the Italian market) using the future markets. This model allow us to cope several of the critical points rises by the literature about the implementation of the PCP to empirical dataset as the inclusion of the dividends and/or the choice of an adequate risk-free rate. These violations are also analyzed with respect to measures of liquidity as the number of contracts. This model is applied to a unique intraday dataset including options, futures and underlying values. The empirical results suggest a quite high level of pricing efficiency in the Italian Option market to be compared to a selected post-crisis period and a weak negative relation between inefficiency and liquidity.

Key words: Index option, pricing efficiency, liquidity

1. Introduction

The Italian Index Option market represents one of the most developed market among European index option market. The growth has been dramatic since the introduction of this markets in November 2005. The usage of index option is particularly important in terms of risk-sharing strategy for mutual fund which use to have portfolio highly correlated to the index market. However the pricing efficiency of the derivatives market has been investigated only by a restricted literature with reference to the Italian Markets, since most of the contributions focus on US derivatives markets.

The most common notion of the pricing efficiency for option markets relies on the existence of arbitrage opportunity. Since the seminal paper by Stoll (1969), the put-call parity relationship (Black and Scholes, 1973) has been the reference point for most of empirical investigation. It has been performed over the years for different markets and sample. This relationship allows to test for the so called cross-market efficiency, which is based on test of the joint efficiency of the option and the underlying asset. The assumption underlying this relationship has also been topic of discussion, but also the difficulties to translate this assumptions to the real dataset in the empirical investigation. A general issue relates to the impossibility to distinguish between the inefficiency driven by the underlying and the inefficiency due to the option market solely. On the other hand, the required data synchronicity, underlying replication, the choice of the risk-free rate, the estimation of the dividends and more recently the determination of the transaction costs have represented issue for several investigation. In the attempt to undergo these difficulties, several extended models has been developed but no one of them can be considered a solution. Most of the literature in the recent years focus on a different notion of efficiency referred to as internal efficiency. Similarly to the previous efficiency notion, this test is pricing model-free, that is to say is not dependent on the specification of any asset pricing model for the valuation of the option. Differently from the PCP, it considers strategies implemented only by assuming position on the option markets (among others box spread strategy). However this notion of efficiency can not be consider as a substitute to the previous more intuitive one, and a complete analysis on the efficiency of the derivatives markets should refers to both analysis.

The paper offers a new general framework for the computation of the violation of the PCP. The new methodology try to overcome to the main shortcomings of the traditional implementation mentioned above. The main feature of this methodology is the use of the future markets having the same underlying as the options. By imposing the well-know pricing condition on the future markets we reach two targets. First, if arbitrage profits emerge from the PCP, this are due solely to option markets inefficiency. Secondly, we exploit the market future price equilibrium relationship to obtain a discount factor which is constant over time, but calibrated for every maturity. This discount factor allows us to generate synthetic future perfectly matching the relative options in the PCP (same strike price). Moreover, this discount factor implicitly collects also the information about the expected future dividends. Regarding synchronicity of the data, the empirical analysis focus on a intra-day dataset kindly provided by Italian Clearing House, Cassa Compensazione e Prestiti (CCP), and select option traded within one minute horizon. Next to the definition of this new framework for the definition of the violations of the PCP, the paper relates to measure of liquidity as bid-ask spread. This measure together with the brokerage commission and the price impact represent the components of the transaction costs. Since the PCP is derived under the assumption of frictionless markets, several works (in particular in the US market) shows that, once the transaction cost are taken into account, the violations emerged by PCP disappears. Given the difficulties associated with the definition of the brokerage fees and price impact, most of the works focus on the bid-ask spread as proxy of the transaction cost. It is also considered as a proxy of the liquidity of a markets and it has been explained by an asymmetric information argument. In particular the bid-ask spread has been explained by adverse selection cost, inventory cost and order processing cost. Among this, the first component seems to be more relevant from a market maker perspective. Intuitively, we expect a negative correlation between the contracts traded and the bid-ask spread: higher is the cost associated with trading, less trades will take place in the markets. However the literature also reports a positive correlation: when the trades increase, the market maker tend to wider the spread. Since the market maker can not distinguish between informed traders and uninformed trades, he prevents himself by increasing the bid-ask spread to be compensated for the trading with informed traders.

Therefore, to investigate the relationship between efficiency and liquidity in the Italian index option markets, we need to estimate the bid ask-spread

associated with every option quotation. Unfortunately these data are not available for the Italian markets. Most of the literature on the Italian markets adopts a given estimates of the bid-ask spread constant over time and over price.

Formally we investigate the relationship between traded contracts and violations to report if there is negative or positive correlation and the relationship between violations and bid-ask spreads to report markets inefficiency if frictions are present in the markets.

2. Methodology

According to PCP relationship (Black and Scholes, 1973), the current price of a call option (C_0) net that of a put option (P_0) is equal to the current value of the underlying asset (S_0) less the present value of the strike price (X) of the options calculated at the risk-free rate (r), with (T) being the residual duration of the contracts:

$$C_0^T - P_0^T = S_0 - Xe^{-rT} \quad (1)$$

At the same time, keeping symbols, the forward value (F_0^T) is the future value of the current price of the underlying, that is

$$F_0^T = S_0e^{rT} \quad (2)$$

Generally speaking, the value of a derivative is the present value of the expected payoff. Therefore, if at time 0 we go for a long future position with (forward) price F_0^T , we face a payoff equal to the maturity spot price S_T less the delivery price F_0^T . In a no-arbitrage setting we take expectations with respect to the risk-neutral probability. A futures price is martingale with respect this probability measure, being $F_0^T = \tilde{E}[S_T]$. Accordingly, the present value is

$$\tilde{FV}_0^T = \frac{\tilde{E}[S_T - F_0^T]}{e^{rT}} = \frac{\tilde{E}[S_T] - F_0^T}{e^{rT}} = 0 \quad (3)$$

With this pricing rule, a speculator is expected to break even when the futures market fairly prices the deliverable asset¹. With no reference to the probability measure, the value of a long forward position is

$$FV_0^T = (F_0^T - X) e^{-rT} \quad (4)$$

¹The implication of the efficient market hypothesis (Fama, 1970) is that the forecast of a

By substituting 2 in 1 we have the equivalence between a portfolio of long call and short put and a long future position:

$$C_0^T - P_0^T = (F_0^T - X) e^{-rT} \quad (5)$$

As known, if the underlying asset pays dividend at a known rate (g), equations 1, 2 and 5 become:

$$Cg_0^T - Pg_0^T = S_0e^{-gT} - Xe^{-rT} \quad (6)$$

$$Fg_0^T = S_0e^{(r-g)T} \quad (7)$$

$$Cg_0^T - Pg_0^T = (Fg_0^T - X) e^{-rT} \quad (8)$$

By equation 7 we can infer the rate g as:

$$g = r - \frac{1}{T} \left(\ln \frac{Fg_0^T}{S_0} \right) \quad (9)$$

Hence, given the risk-free rate and the equilibrium in the future market, the dividend growth rate can be derived and applied to the option prices. Alternatively, given the dividend growth rate, the discount rate can be derived.

A full equilibrium across option and future markets is guaranteed by equations 5 and 8. To apply the quoted equivalence strikes prices of options and future price have to be equal. Therefore if we take the option prices as standpoint, the *corresponding* future value (FV_0^T) can be inferred either from the algebraic summation of a long call and a short put (see equations 5 and 8) or from equation 4 where X has to be consistent across options and future. In this last case, when the strike are not consistent across the two derivative markets, we can reconstruct a *synthetic* future value (SFV) with the same strike of the options by calculating the present value of the expected future price and the selected strike that is

$$SFV_0^T = (E[S_T] - X) e^{-rT} \quad (10)$$

final price, $E^t[S_{t+1}|\Omega_t]$ is unbiased, which means that on average, the expected price equals the actual price. Another way to describe market efficiency is the so called orthogonality condition, which uses the fact that the forecast error should be on average zero and uncorrelated with the information contained in Ω_t .

where $E[S_T]$ is given by equation 2 or 7 according to the absence or presence of the dividend yield. As a consequence, the equilibrium can be appraised by means of:

$$C_0^T - P_0^T - (S_0 - Xe^{-rT}) = 0 \quad (11)$$

or, in case of dividend, by means of:

$$Cg_0^T - Pg_0^T - (S_0e^{-gT} - Xe^{-rT}) = 0 \quad (12)$$

Whenever formulations 11 and 12 are different from zero there is an arbitrage opportunity and the markets are not in equilibrium. Consistently the larger the difference in absolute value (with no reference to the sign), the more relevant the equilibrium violation. Moreover, the (in)efficiency can be linked even to a liquidity measure of the markets by evaluating the relationship between violation and the number of the contracts.

The model can also be easily extended to consider the effect of the bid-ask spread when the differential is not explicit in the prices of the options. In particular, the spread implies that long (short) prices are higher (lower) than fair ones. Therefore, where bid-ask differentials are directly available, the model can be implemented by using appropriate values. At the same time, if the differentials are not available, the difference between call and put has to be eventually adapted by a fix percentage able to synthesize the average differential.

3. The dataset

Since the research is based on the equation 12, relevant variables are the call and put option prices, the current value of the underlying, as well as the dividend and the interest rates. As seen in section 2, the dividend rate g is to estimate through the futures markets, by means of equation 9. To obtain the relevant variables the dataset is composed by intra-day call and put option prices as well as intra-day future prices on the Italian stock index S&P/MIB. In addition to this underlying intra-day quotes have been used. As far as the interest rates are concerned the 1-month Euribor rate has been selected, in order to preserve consistency between the duration of the derivative contracts and the time horizon of the risk free rate. As far as measures are concerned, consistent measurement has been selected and annualized continuous rates have been implemented. The Italian Clearing House (<http://academy.borsaitalia.it>) has provided the prices dataset,

while values of the Euribor have been downloaded from the official website database (<http://www.euribor.org>).

Time period under observation goes from the 1st of September to the 30th of November of 2007. The horizon under investigation is particularly worth of investigation because it allows to compare the efficiency and liquidity conditions of the Italian index option markets before and after the recent financial crisis. We adopt as reference point for the starting time of the crisis the moment from which news about the financial difficulties about Northern Rock impacted to the Italian stock index.

Regarding the options, the dataset includes for each options: the negotiation hour, the clearing hour, the type, the maturity, the strike, the option price (expresses in index point, each equal to 2.5), number of contracts and quantity of options traded. These options are European-style option contracts. Everyday six different expiration dates are quoted: four quarterly (March, June, September and December) and two monthly (the nearest two months). The expiration date is the third Friday of the expiration months and the strike prices have fixed increments of 500 index point and everyday at least nine different strikes for each expiration are quoted. From this main dataset we extract the couples of options composed by put and call having the same maturity, strike price and corresponding negotiation time within the same minute in the same day. Once these couples are obtained we matched them with future having the same maturity and traded in the same negotiation day and within the same minute as the options couples. Finally we matched the option couples and futures with the underlying values. After imposing these filters, our dataset is composed by about 3000 couples of options and corresponding values for futures and options markets.

On this dataset we worked as previously discussed (see section 2) in order to gain the time series of the dividend growth and then the time series of the violation of the equilibrium (equation 12). The absolute value of the violation have been analyzed with reference to a measure of the liquidity of the markets by investigating the correspondence between the absolute vale of the difference and the number of contracts corresponding to the transactions on the option market (summation of call and put contracts). Another relevant information is given by the ratio of the violation in absolute value to the strike prices of the corresponding option transaction. The absolute value of the violation states also a sort of maximum level of transaction costs that the markets are able to sustain. The ration of such value to the strike gives a percentage of sustainability. Finally both the series are then analyzed with

Table 1: Descriptive statistics of absolute and sign values of violations

statistic	absolute value	sign value
mean	162,76	2,87
standard error	5,94	6,75
median	10,17	-0,10
mode	5,19	-9,84
standard deviation	301,32	342,48
sample variance	90796,46	117289,94
curtosis	14,34	12,41
skewness	3,02	0,05
range	2970,88	5884,20
minimum	0,00	-2913,32
maximum	2970,88	2970,88
sum	418623,31	7374,11
observation	2572,00	2572,00

reference to the quantity of contracts.

4. Results

The analysis, whose results are summarized by following tables and charts, provides us with an interesting picture of the Italian option market. Two main series are under investigation: the absolute values of the violations (with and without signs) and the relative value calculated as the ratio of the values to the strike. Descriptive statistics for the series are reported by the tables 1 and 2. As can be easily appreciated, the majority of violation are of small size and concentrate around the zero value, both for the absolute and the relative evaluation. A synthetic indicator of the relative measure of the violation can be the ratio of the mode of the distribution to the range interval. Such ratio is approximately equal to $|0,17\%|$ for the absolute values and to $|0,13\%|$ for relative values. Therefore, the Italian option markets seems to be quite efficient at least in the period we approach in this paper. This results is confirmed by the analysis of the distribution of violations, which confirms that there is a prevailing equilibrium in the market. As far as the connection with liquidity is concerned, we can observe that the relationship is not strong. The correlation coefficient is not significant and it is near to

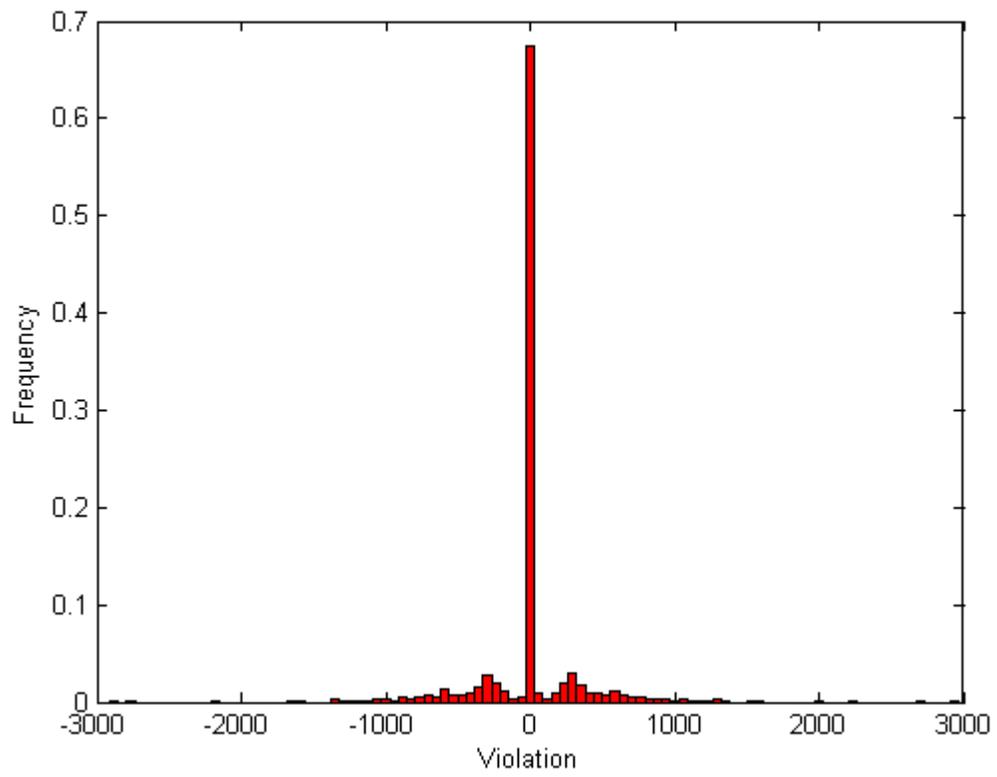


Figure 1: Distribution of violations

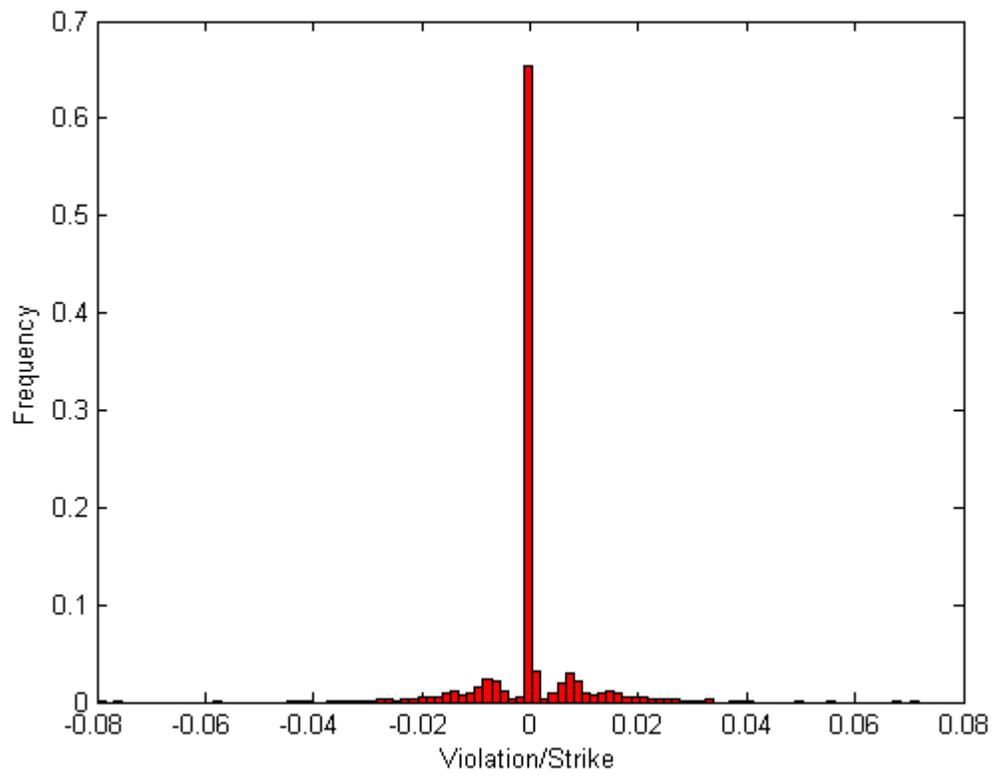


Figure 2: Distribution of relative violations

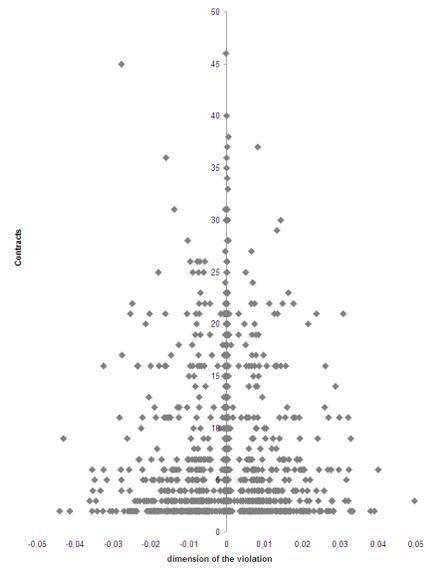
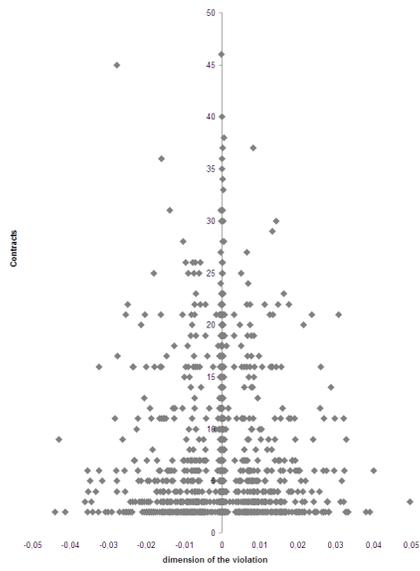
Table 2: Descriptive statistics of relative value of violations $\frac{viol}{strike}$

statistic	value
mean	0,041%
standard error	0,02%
median	0,03%
mode	0,01%
standard deviation	0,77%
sample variance	0,01%
curtosis	1518,97%
skewness	308,71%
range	7,98%
minimum	0,00%
maximum	7,98%
sum	10,66
observation	2572,00

the null value, as can be appreciated also by scatterplots linking the number of contracts of the single transaction to the two measures of violations. The scatterplots show also that the number of violations is a decreasing function of the number of contracts, thus stressing that the depth of the derivative markets has a great influence on the pricing system. A part from the numerical results, there is an important methodological issue. The suggested approach is relatively easy to implement in any market and can serve for space and time comparison. Therefore, we can easily expand the analysis in time and test the effect of special and critical event of the pricing efficiency of the option market.

5. Conclusion and future research prospects

The paper proved a methodology for testing option pricing efficiency in a cross-market approach. Moving from the general arbitrage relationship for the index option markets represented by the put-call parity, this new methodology exploit the relationship between the future markets and the index option markets and their relative pricing conditions to overcome the difficulties arising from the implementation of these theoretical relationships to empirical datasets. Moreover the model results to be of simple implemen-



tation and can be potentially applied to all the financial markets having a developed market of options and futures on the same index stock as underlying as in the main European countries. From the implementation of the model results that the Italian Index Option market has a quite high level of pricing efficiency. With respect to the number of contract, the relationship between this and the violations has a low and negative value of the correlation. However this analysis can also be improved by considering some aspects. First of all the analysis will be extended to consider a post-crisis period. This will give us an insight on the dynamics of the violations/efficiency between the two periods and it will be possible a comparison of the results in an ante and post-crisis period as in an event study. As in some contribution in the literature, the analysis can also be extended to consider an execution time delay, which consent to investigate how and if the violations of the efficiency persist after one minute. Moreover, given the investigation horizon of one minute, intraday data of the interest rates (as available on Liffe) could provide an even better definition of the expected dividend yield to be taken in to account. On another hand, the current unavailability of data limits our analysis on the interconnection between liquidity and efficiency to the number of contracts. However a full analysis of this relationship would be possible if the bid-ask spread and bid and ask volumes for each transaction would be available.

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