Does the Value at Risk capital requirement sit well in the Solvency II framework?

Abstract
In this paper the Solvency II VaR-based capital requirement is analyzed and discussed. The new European risk-based system of prudential regulation for insurers could in fact increase, not decrease, the fragility of the insurance industry. More specifically, the VaR capital requirement exposes insurance companies to a potentially huge systemic effect, as the bigger-better diversified insurers have high default probabilities in case of market shortfalls. This paper shall suggest and discuss some adjustments to the current Solvency II framework.

Keywords: Solvency II, Value at risk, Prudential regulation, Capital requirements
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1. Introduction

The insurance industry has only been marginally impacted by the financial crisis and the few insurers that experienced serious difficulties were brought down by non-insurance activities. The relative strengthening of the insurance sector compared to the systemic crisis of the banking system is due to different factors. Among these, life insurance products often shift investment risks to the policyholder and in non-life insurance products, the insurer assumes diversifiable risk quite exclusively. A common key factor is less exposure to systematic risk which increase the insurance system’s resilience to a global systemic crisis.

Furthermore, the Solvency II European Directive (2009/138/EC) is to introduce a risk based approach to capital requirements as from 2014. More specifically, the directive prescribes that the Solvency Capital Requirement “shall correspond to the Value-at-Risk of the basic own funds of an insurance or reinsurance undertaking subject to a confidence level of 99.5% over a one-year period”. Apparently, the new Solvency II framework is a step towards a more resilient insurance system seeing that the principle of more risk - more capital provides the right incentives to better risk management practices while lowering the risk - reward profile.

Unfortunately, as this paper demonstrates, this is not the case. The Solvency II framework could increase the insurance system’s probability to be involved in a global systemic crisis thus contributing to the crisis itself. Even if the principles and the overall Solvency II structure are economically viable, they fail when the wrong risk measure - the Value at Risk - is applied to the correct principle of more risk – more capital. The simple model developed in this paper - based on a frictionless and normally distributed world - demonstrates that the Value at Risk measure is incorrect, simply because it is a total risk measure. Nonetheless, the financial economics literature reminds us that there are two
sharply different types of risk: systematic risk and diversifiable risk. Broadly speaking, two insurance companies with the same total risk but with different compositions between systematic risk and diversifiable risk have, according the Solvency II directives, the same Solvency Capital Requirement. The more diversified the company is (i.e. the more exposed to systematic risk), the more exposed to market shortfalls it is. Therefore, in the case of a global systemic crisis, it has a higher probability for bankruptcy. Under the Solvency II framework, there are clear incentives for diversification, growth in size (which implies more diversification) and systematic risk assumption. Under the Solvency II regimes, the bigger - better diversified insurance companies could have a higher probability to be involved in the next global crisis.

This paper contributes to the insurance economics literature, exploiting the relationship between risk-reward profile, default probability and systematic vs. diversifiable risk, in a VaR capital requirement framework. Even if it is based on a very simple theoretical model, this paper exposes a large pitfall in the Solvency II directives and suggests some means to correcting it.

The financial economics literature has only recently indicated the drawbacks of diversification for financial institutions (Ibragimov et al., 2011, Golstein and Pauzner, 2004; Allen and Gale, 2005; Allen and Carletti, 2006). Wagner (2010) noted that “even though diversification reduces each institution’s individual probability of failure, it makes systemic crises more likely”. In this paper, results are shown that could also be extended to the insurance sector. In addition, within the Solvency II framework a marked inducement to diversification emerges and this makes the insurance sector more vulnerable to systemic risk.
The new regulatory framework for insurance companies has been widely discussed and sometimes criticized (cf. among others Ashby, 2011, Doff, 2008, Eiling et al., 2007, Eiling and Schmeiser, 2010). Hereafter, a new specific and original critique of the Solvency II framework emerges. The Solvency II regime uses an inadequate risk measure to compute the Solvency Capital Requirement.

The VaR measure is often criticized since only in a normally distributed world it has desirable properties such as subadditivity (cf. Artzner et al., 1999. However, Dhaene et al., 2008, argue that the VaR could be a good risk measure for regulatory purposes due to the coherent risk measures that could be “too subadditive”). Additionally, it does not adequately consider what happens in the scenarios exceeding the VaR measure (cf. Dowd and Blake, 2006, Alexander and Baptista, 2006, Basak and Shapiro, 2001). In this paper, the Value at Risk is suboptimal for regulatory purposes in view of the fact that it is a total risk measure and from this point of view, the other coherent risk measures suffer the same drawbacks too. Here the central point of the model is the distinction between systematic risks and diversifiable risks which implies that a VaR capital constraint, which does not consider this distinction, leads to some undesired side effects.

Lastly, the effects of a VaR capital constraint have already been analyzed in literature for banks (cf., among others, Leippold et al., 2006, Cuoco and Liu, 2006). As of yet, no similar studies exist for insurance companies. This gap is filled by introducing a very simple, but complete model applicable to the insurance industry.

The layout of this paper is as follows: In the next section the model, based on a perfect normally distributed world, is introduced and analysed. Clearly, if the VaR based capital requirement framework fails in a perfect and normally distributed world, no better performance is expected if applied in true insurance markets, which have frictions and highly
asymmetric distributions. In the final section, the policy implication of the model is discussed and some suggestions for Solvency II regulation improvement are introduced.

2. The model

2.1. The insurance company structure and its activities

Consider an insurer that operates in one single period and in a perfect and normally distributed financial market. The insurance company subscribes a portfolio of insurance contracts at time $t=0$ which gives a random cash outflow for claims and expenses equal to $\tilde{L}_1$ at time $t=1$. $\tilde{L}_1$ is normally distributed with the expected value $E(\tilde{L}_1)$ and the standard deviation equal to $\sigma(\tilde{L}_1)$. The sum that the insurer should set aside at time $t=0$ to fulfil the insurance obligation, i.e. the technical provision, is denoted with $L_0$. Under the Solvency II framework, which is of interest here, a fair value measure for technical provisions is required (“the value of technical provisions shall correspond to the current amount insurance and reinsurance undertakings would have to pay if they were to transfer their insurance and reinsurance obligations immediately to another insurance or reinsurance undertaking”, art. 76.2 EC directives 138/2009). Practically, if random cash outflows are replicable by marketable financial instruments, technical provisions are equal to the value of the replicating portfolio. For non replicable cash flows, technical provisions are equal to the best estimate plus a risk margin computed through the cost of capital approach (see Floreani, 2011 for the rationale of this method in a financial economics framework). For convenience, only the replicable cash flows case is addressed hereinafter. Therefore $L_0$ is the market value of the $\tilde{L}_1$ replicating portfolio.

The sum $A_0 = L_0$ is invested in financial instruments that give at time $t=1$ a random cash inflow equal to $\tilde{A}_1$, which is again normally distributed with $E(\tilde{A}_1)$ and standard deviation equal to $\sigma(\tilde{A}_1)$. $A_0$ is the value of assets to cover technical provisions. The cash
flows are not necessarily invested in the $L_1$ replicating portfolio. However, in this situation, that is if $\bar{A}_1 = \bar{L}_1$, the insurer has immunized its asset-liability risk. In normal situations $\bar{A}_1 \neq \bar{L}_1$ and some asset-liability risks emerge.

The insurer has an additional amount $F_0$, named free assets, which is again invested in marketable financial instruments that give the normally distributed random cash inflow $\tilde{F}_1$ at time $t=1$. The insurance company net cash flows at the end of the period are:

$$\tilde{S}_1 = \bar{A}_1 + \tilde{F}_1 - \bar{L}_1$$

which is, due to the normal distribution properties, once again normally distributed, whose value at time $t=0$ is named net asset value and is equal to:

$$S_0 = A_0 + F_0 - L_0 = F_0$$

Let us assume that all financial assets are negotiated in frictionless and complete markets whose returns follow the following market model:

$$\tilde{r}_i = E(\tilde{r}_i) + \beta_i [\tilde{r}_m - E(\tilde{r}_m)] + \tilde{\epsilon}_i$$

for each financial instrument $i$ \[1.\]

where $\beta_i$ and $\tilde{\epsilon}_i$ are respectively, the beta coefficient and the specific risk of the financial instrument $i$ and $\tilde{r}_m$ is the return on market portfolio\[1\].

Under this framework, if no arbitrage opportunities on the financial market are allowed, the Capital Asset Pricing Model could be assumed to hold, i.e.:

$$E(\tilde{r}_i) = r_f + \beta_i [E(\tilde{r}_m) - r_f]$$

for each financial instrument $i$ \[2.\]

where $[E(\tilde{r}_m) - r_f]$ is the market risk premium.

Clearly, equations [1] and [2] also hold for the covering asset portfolio (A), the free asset portfolio (F) and the replicating portfolio (L). In this situation the net asset cash flow ($\tilde{S}_1$) is normally distributed with\[2\]:

\[1\] In the classic market model [1], it is assumed that $E(\tilde{\epsilon}_i) = 0$, $E(\tilde{\epsilon}_i \tilde{\epsilon}_j) = 0$ and $E(\tilde{\epsilon}_i \tilde{r}_m) = 0$. 

\[2\]
\[
\tilde{S}_1 \sim \varphi \left( E(\tilde{S}_1), \sigma(\tilde{S}_1) \right) = \varphi \left( S_0 \cdot E(\tilde{r}_S), S_0 \cdot \sigma(\tilde{r}_S) \right) \tag{3}
\]

\[
E(\tilde{r}_S) = r_f + \beta_S [E(\tilde{r}_m) - r_f] \tag{4}
\]

\[
\sigma(\tilde{r}_S) = \sqrt{[\beta_S \cdot \sigma(\tilde{r}_m)]^2 + \sigma^2(\tilde{e}_S)} \tag{5}
\]

\[
\beta_S = \beta_F + \frac{L_0}{S_0} \cdot (\beta_A - \beta_L) \tag{6}
\]

\[
\sigma(\tilde{e}_S) = \sqrt{\sigma^2(\tilde{e}_F) + \frac{L_0^2}{S_0^2} \cdot \left( \sigma^2(\tilde{e}_A) + \sigma^2(\tilde{e}_L) \right)} \tag{7}
\]

\(\beta_S, \beta_F, \beta_A\) and \(\beta_L\) are the net assets, free assets, covering assets and replicating assets beta coefficients respectively, and \(\tilde{e}_S, \tilde{e}_F, \tilde{e}_A\) and \(\tilde{e}_L\) are the net assets, free assets, covering assets and replicating assets specific risks respectively. The symbol \(E(.)\) and \(\sigma(.)\) indicate the expected value and the standard deviation operators. The symbol \(\sim\) means “distributed as” and the function \(\varphi(\mu, \sigma)\) is a normal distribution function with the mean \(\mu\) and the standard deviation \(\sigma\).

In this model, the expected net asset return is only affected by the systematic risk \(\beta_S\), the only risk priced in a frictionless world. The net asset systematic risk is affected (cf. Eq. [6]) by the free asset systematic risk \(\beta_F\), the mismatch between the asset-liability systematic risk, i.e. \((\beta_A - \beta_L)\), and by leverage ratio \(\frac{L_0}{S_0}\), which is a multiplier of the asset-liability risk. The total risk on the net asset (cf. Eq. [5]) is not only affected by systematic risk but also by the diversifiable asset specific risk. The latter (cf. Eq. [7]) depends on the free asset specific risk \(\sigma(\tilde{e}_F)\), the covering asset specific risk \(\sigma(\tilde{e}_A)\), the replicating portfolio specific risk \(\sigma(\tilde{e}_L)\) and the leverage ratio \(\frac{L_0}{S_0}\).

At time 0 the insurance company could be assumed to be involved in the usual insurance company activities:

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2 Equations [3.]-[7.] are obtained with simple algebra, assuming that the market model and the CAPM hold. This and all subsequent mathematical derivations, which are omitted or summarized hereinafter, are available upon request by the author.
- The direct insurance and the insurance risk management activities;
- The investment and financial risk management activities.

The direct insurance activity involves subscribing to the insurance contract portfolio. Given the insurance contract portfolio \( \tilde{L}_1 \) emerging from the direct insurance activity, the insurance risk management activity is devoted to acting in indirect insurance markets (for example reinsurance) in order to obtain the net insurance portfolio \( \tilde{L}_1 \). Although we have assumed that financial markets are frictionless, complete and without arbitrage opportunities and that insurance cash outflows may be replicated by marketable securities, no hypothesis is required on the direct and indirect insurance markets between the insurer and policyholders or between the insurer and the reinsurer. Even if the analysis of the reinsurance activity is of interest, this issue is not addressed in this paper as only the (net) insurance portfolio is considered hereinafter. Therefore, the insurance activities are summarized by \( P^* \), the (net) premium obtained in \( t=0 \) for the portfolio \( \tilde{L}_1 \), and by the portfolio characteristics \( E(\tilde{L}_1) \), \( \beta_L \) and \( \sigma(\tilde{L}_1) \), which are proxies for the portfolio size, the systematic liability risk and the diversifiable (but not diversified) insurance risk respectively.

The value of technical provisions is \( L_0 = E(\tilde{L}_1)/[1 + E(\tilde{r}_L)] \), where according to CAPM, \( E(\tilde{r}_L) = r_f + \beta_L [E(\tilde{r}_m) - r_f] \).

The investment and financial risk management activities are aimed at managing investments in order to reach the desired risk-reward profile. In our model, the risk – return profile on net assets is univocally determined by the systematic risk on net assets \( \beta_S \) and the insurer is able to act on \( \beta_A \), \( \beta_F \) or the leverage ratio \( \frac{L_0}{S_0} \) in order to reach the desired risk return-profile.
To complete this model, it should be considered that if the insurance company’s top management operates in the shareholders’ interests, owing to the shareholders’ limited liability, the sum given to shareholder at time $t=1$ is:

$$\tilde{E}_1 = MAX[\tilde{A}_1 + \tilde{F}_1 - \tilde{L}_1; 0] = MAX[\tilde{S}_1; 0] \quad [8.]$$

In a perfect and normally distributed world, it is relatively simple to compute the equity value. For instance, following Brennan (1979) eq. [39], we obtain:

$$E_0 = S_0 \cdot N(d) + \frac{\sigma(\tilde{S}_1) \cdot n(d)}{r_f} \quad [9.]$$

$$d = \frac{r_f \cdot S_0}{\sigma(\tilde{S}_1)} \quad [10.]$$

where $N(.)$ and $n(.)$ are standard normal cumulative and density functions, respectively. The shareholders’ risk-return profile depends on the equity beta which is equal to:

$$\beta_E = \beta_S \cdot N(d) \quad [11.]$$

The equity risk–return profile is lower than the net asset risk-return profile ($N(d) \leq 1$), seeing as the limited liability protects shareholders from loss in the case of default.

The expected return on equity is:

$$E(\tilde{r}_E) = \frac{E(\tilde{E}_1)}{E_0} = r_f + \beta_E [E(\tilde{r}_m) - r_f] \quad [12.]$$

In frictionless and perfect competitive financial and insurance markets, the premium $P^*$ obtained for insurance contract portfolio $L_1$ should be equal to:

$$P^* = P_0 = L_0 - (E_0 - S_0) \quad [13.]$$

where $P_0$ is the value of insurance liabilities considering the shareholders’ limited liability. In this perfect competitive situation there is no opportunity for shareholder value creation from insurance or financial activities and the risk–reward profile which best reflects shareholders’ interests could be assumed to be the objective that is pursued by top management.
If insurance companies are able to gain some extra profit (or extra loss) from insurance activities, i.e. the premium effectively raised is $P^* \neq P_0$, the amount that shareholders should invest at time 0 in the insurance company is $I_0 = E_0 - P^* + P_0$ and the expected return on shareholder investment could be greater (or less) than $E(\hat{r}_E)$, i.e.:

$$E(\hat{r}_I) = \frac{E(E_{\hat{r}_I})}{I_0} = E(\hat{r}_E) \cdot \frac{E_0}{I_0} = \left[ \frac{P^* - P_0}{E_0} + \beta_E \left( E(\hat{r}_m) - r_f \right) \right] \cdot \frac{E_0}{I_0} \quad [14.]$$

In this situation, the objective for top management - which acts in shareholders’ interests - could be complex but quite realistic. Top management should exploit all insurance business opportunities while considering the risk-reward profile which emerges from investment and financial risk management activities. Hereinafter, no explicit management objective functions are introduced except for a classic non-satiety and risk aversion hypothesis. More specifically, it could be assumed that top management prefers a higher $E(\hat{r}_I)$ given the systematic and/or total risk, or prefers less systematic and/or total risk given $E(\hat{r}_I)$.

Lastly, it can be observed that, in the absence of regulation, the insurer depicted in this model may be able to act with a very high leveraged financial structure (which implies a very high default probability) and, if direct insurance markets are not perfectly competitive, it is also possible that the insurers operate with no shareholder investments. This is the case, for example, if $P^* \geq L_0$. However, in this situation the insurers’ default probability could be very high (for example, if $P^* = L_0$, $I_0 = 0$ and asset-liability risk is absent, the default probability is equal to 50%).
2.2. Solvency Capital Requirement based on the Value at Risk measure

In this section, the Solvency Capital Requirement based on the Solvency II directive is introduced. More precisely, Solvency II requires a Solvency Capital Requirement (SCR) that prevents default in 99.5% of cases over a one-year time horizon.

Formally, the insurer default probability at time t=1 is:

$$\text{Prob}(\tilde{S}_1 \leq 0) = \text{Prob}(\tilde{r}_s \leq 0) = p = N\left[-\frac{E(\tilde{S}_1)}{\sigma(\tilde{S}_1)}\right] = N\left[-\frac{E(\tilde{r}_s)}{\sigma(\tilde{r}_s)}\right]$$  \[15.\]

where \(N(.)\) is the standard normal cumulative function.

Therefore, the Solvency Capital Requirement based on a Value at Risk measure with a default probability \(p\) and a level of confidence \(1-p\) is the minimum level of net asset value \(S_0^*\) (or the minimum level of free asset \(F^*_0\)) that should be invested in order to prevent insolvency with probability \(p\). Given the normal distributions of asset and liability, the Solvency Capital Requirement is the minimum level of net asset value \(S_0^*\) such that:

$$E(\tilde{S}_1) - N^{-1}(1-p) \cdot \sigma(\tilde{S}_1) \geq 0$$  \[16.\]

where \(N^{-1}(\cdot)\) is the inverse of the standard normal cumulative function.

In order to obtain a relatively simple analytic solution for \(S_0^*\), it could be useful to assume, without loss of generality, that the assets to cover the technical provisions are invested in order to immunize asset-liability risk, i.e. \(\beta_A = \beta_L\). In this situation, the insurer risk-reward profile is univocally defined by the investments of free assets, i.e. \(\beta_S = \beta_F\), and the Solvency Capital Requirement is\(^3\):

\(^3\) Considering equations from [3] to [7], equation [16.] becomes:

$$r_f + \beta_F \cdot [E(\tilde{r}_m) - r_f] - N^{-1}(1-p) \sqrt{\left(\beta_F \cdot \sigma(\tilde{r}_m)\right)^2 + \sigma^2(\tilde{r}_F) + \frac{L_0^2}{S_0^2} \left(\sigma^2(\tilde{r}_d) + \sigma^2(\tilde{r}_L)\right)} \geq 0$$

Considering that the first two addendum and the quantities into the square root are positive:

$$(r_f + \beta_F \cdot [E(\tilde{r}_m) - r_f])^2 - \left(N^{-1}(1-p) \cdot \beta_F \cdot \sigma(\tilde{r}_m)\right)^2 - \left(N^{-1}(1-p) \cdot \sigma(\tilde{r}_F)\right)^2 \geq \frac{L_0^2}{S_0^2} \left(\sigma^2(\tilde{r}_d) + \sigma^2(\tilde{r}_L)\right) \ast$$

The minimum level of net asset value \(S_0^*\) that satisfies the inequality \([16.]\), could be obtained by \([\ast]\) with simple algebra.

\[11\]
\[
S^*_0 = L_0 N^{-1}(1 - p) \frac{\sigma^2(\xi_A) + \sigma^2(\xi_L)}{(r_f + \beta_F \cdot \left[ E(\tilde{r}_m) - r_f \right])^2 - (N^{-1}(1-p) \cdot \beta_F \cdot \sigma(\tilde{r}_m))^2 - (N^{-1}(1-p) \cdot \sigma(\xi_F))^2} \tag{17}
\]

A brief discussion of the formula could be of interest here.

Firstly and intuitively, the minimum required solvency ratio \(S^*_0/L_0\) is greater when:

- the confidence level \((1-p)\) is higher (or the maximum tolerable default probability level \(p\) is lower); in Solvency II the confidence level is 99.5\% \((p=0.5\%)\);
- the diversifiable (but not diversified) risks on assets and liabilities are higher;
- the insurer exposure to systematic risks \((\beta_F)\) is higher\(^4\).

Secondly, when the minimum solvency ratio \(S^*_0/L_0\) is set, a clear trade-off between systematic risks \((\beta_F)\) and diversifiable risks \((\sigma(\tilde{x}_A), \sigma(\tilde{x}_L)\) and \(\sigma(\xi_F)\)) emerges. In particular, an increase in \(\beta_F\) requires a reduction in some diversifiable risks in order to preserve the same minimum required solvency ratio \(S^*_0/L_0\).

Third, given the liability portfolio characteristics \((E(\tilde{L}_1), \beta_L\) and \(\sigma(\xi_F)\)), the lowest level in the Solvency Capital Requirement (or in the minimum required solvency ratio \(S^*_0/L_0\)) is obtainable if the financial risk management activities are capable of completely eliminating asset risks and asset-liability risks, that is \(\beta_A = \beta_L, \beta_F = 0, \sigma(\tilde{x}_F) = 0, \sigma(\xi_A) = 0\). In this situation, the Solvency Capital Requirement is equal to:

\[
S^*_0 = \frac{N^{-1}(1-p) \cdot L_0 \cdot \sigma(\xi_L)}{r_f} \tag{18}
\]

Clearly, the expected return on surplus here is in line with the return on risk free asset. If the insurance company wants to reach a higher risk-reward profile it should assume systematic risk.

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\(^4\) This latter point is only effective for plausible values of the relevant parameters. Under particular market conditions, a marginal increase in betas could decrease the Solvency Capital Requirement as the increase in expected net assets overwhelms the effects of the increase in standard deviation.
Lastly and most importantly, the introduction of a VaR capital requirement establishes an important restriction on the assumption of systematic risk and consequently on the risk–reward profile that is feasible. Actually, it could be demonstrated that the superior limit for systematic risk assumption is

\[ \beta_S^* = \frac{r_f}{N^{-1}(1-p) \cdot \sigma(\bar{r}_m) - [E(\bar{r}_m) - r_f]} \]  

[19.]

This implies that the maximum feasible expected return on equity is:

\[ E(\bar{r}_E^*) = \frac{E(\bar{E}_1)}{E_0} = r_f + \beta_S^* \cdot N(d) \cdot [E(\bar{r}_m) - r_f] \]  

[20.]

An insurance company is able to reach this superior limit only if it is fully diversified. In particular, this upper limit can be reached if the ratio between systematic risk and total risk tends to 1. Let the diversification degree (DG) be equal to:

\[ DG = \frac{\beta_S^* \cdot \sigma(\bar{r}_m)}{\sigma(\bar{r}_S)} = \frac{\beta_S^* \cdot \sigma(\bar{r}_m)}{\sqrt{[\beta_S^* \cdot \sigma(\bar{r}_m)]^2 + \sigma^2(\bar{E}_S)}} \]  

[21.]

If DG→1, then the maximum systematic risk exposure tends to be equal to \( \beta_S^* \).

The introduction of the Solvency Capital Requirement may or may not affect the management of the insurance company. It is not affected if, given insurance portfolio \( L_1 \) and available own funds \( F_0 \), the risk-reward profile required by shareholders could be reached without infringing on the Solvency Capital Requirement constraints. Otherwise, the insurer is affected by regulation. There are two different possible situations. On the one hand, the risk-reward required by the shareholder is feasible since it is lower than \( E(\bar{r}_E^*) \) in eq. [20]. However, the insurer is required to increase the diversification degree in eq. [21]. This could be done either through a reduction in diversifiable asset risks (\( \sigma(\bar{E}_A) \) and \( \sigma(\bar{E}_F) \)) or

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\( ^5 \) In equation [17], the quantity in square root should be positive. Taking into account that the numerator is always non negative, this implies that the denominator should be positive, i.e.:

\[ (r_f + \beta_P \cdot [E(\bar{r}_m) - r_f])^2 - (N^{-1}(1-p) \cdot \beta_P \cdot \sigma(\bar{r}_m))^2 - (N^{-1}(1-p) \cdot \sigma(\bar{E}_P))^2 > 0 \]

Therefore if free assets are invested in a fully diversified portfolio (\( \sigma(\bar{E}_P) = 0 \)), equation [19.] is obtained with simple algebra. Clearly, if the free asset portfolio is not fully diversified, the systematic risk that the insurance company is able to achieve lowers.
diversifiable liability risks ($\sigma(\tilde{\varepsilon}_L)$) or, less likely still, by increasing its own funds. On the other hand, the risk-reward required by the shareholders may not be reached since it is equal or higher than $E(\tilde{\gamma}_E^*)$ in eq. [20]. In this situation, the management of the insurance company has a marked inducement towards fully diversified investments and insurance activities (i.e. to reach a diversification degree close to 1) obtaining the highest feasible risk-reward profile.

At this stage it could be useful to introduce a simple numerical specification:

- Market conditions: risk free rate = 4% ($r_f=1.04$); market risk premium = $[E(\tilde{r}_m) - r_f] = 6%$; market risk volatility = $\sigma(\tilde{r}_m) = 20%$;
- Insurance contract portfolio : $E(L_1)=100$; $\beta_L=-0.1$, $\sigma(\tilde{\varepsilon}_L)=8\%$ (this implies that $L_0=A_0=96.7118$);
- Investments covering technical provisions are fully diversified ($\sigma(\tilde{\varepsilon}_A)=0$) and asset-liability risk is immunized ($\beta_A=-0.1$);
- Available free assets = $F_0=S_0 =20$; free assets are invested in a fully diversified portfolio ($\sigma(\tilde{\varepsilon}_F)=0$) with $\beta_F$ set in order to reach, where possible, the risk-reward profile required by shareholders.
- Regulators require a Solvency Capital Requirement based on a Value at Risk measure with a confidence level equal to 99.5%; therefore, the superior limit for systematic risk (eq. [19.]) is $\beta_D^* = 2.28488$ which corresponds to a maximum shareholder rate of return on equity equal to 17.55% ($E(\tilde{\gamma}_D^*) = 1.1755$).

If the shareholder required rate of return on equity is low (for example 6%), the optimal risk-reward profile may be reached by simply investing the free assets in a fully diversified portfolio with $\beta_F=0.336$. The Solvency Capital Requirement here is 19.0538, less than the available net assets ($S_0 =20$).
Should the shareholder require a higher rate of return on equity (for example 14%), the insurer is to invest its free assets in a fully diversified portfolio with $\beta_F=1.682$. However, the Solvency Capital Requirement is 26.8513 and to fulfil it new own funds of 6.8513 have to be raised or, more likely, part of the insurance portfolio risk needs to be diversified (for instance $\sigma(\bar{c}_L)$ from 8% to 5.959%).

Where the shareholder rate of return on equity is higher than feasible (for example 22%), the maximum risk–reward profile (17.55%) could be reached raising an unlimited quantity of own funds (and investing such a sum in a fully diversified portfolio with $\beta_F=2.28488$) or, more realistically, to fully diversify the insurance contract portfolio ($\sigma(\bar{c}_L)=0$).

In this example, the shareholder return on equity (eqs. [12.] and [19.]) is considered. If direct insurance markets are perfectly competitive, the expected return on shareholder investments (eq. [14]) is equal to the expected return on equity. Whereas, if direct insurance markets are not perfectly competitive, i.e. $P^* \neq P_0$, the incentive to diversification may be lower, in particular when some extra profit is allowed ($P^*>P_0$). Intuitively, if the insurance activity permits value creation, expected return on shareholder investment may be optimized through the insurance activity and the investment or financial risk management activities could be devoted lowering unnecessary risks (diversifiable investment risks) to the minimum level while assuming a not too high systematic risk.

As a simple example, let us consider the previous numerical specification, while allowing an extra profit from insurance activities of five for every 100 expected claims at time 1. For an insurance portfolio with $E(L_1)=100$; $\beta_L=-0.1$, $\sigma(\bar{c}_L)=8\%$ and without investment and asset-liability risks, the Solvency Capital Requirement is equal to 19.1625, the fair value of the insurance portfolio ($P_0$) is 96.7000, the premium raised is 101.7000 ($P^* =$
P_0+5), the minimum capital invested by the shareholder which supports the SCR is 14.1743 and the expected rate of return on shareholder investment is 40.69% \( (E(\bar{\eta}) = 1.4069, \text{cf. eq. [14. ]}) \). With the same insurance activities and some systematic investment risks, for example \( \beta_F = 1 \), a SCR equal to 20.5051 would be required, with a minimum shareholder investment of 15.5257 and the expected rate of return on shareholder investment increases to 45.366%. However, for higher systematic risk exposure (for example \( \beta_F = 2 \)), the minimum shareholder investment is 32.4528 and the expected rate of return on shareholder investment drops to 33.727%.

This example illustrates how in a profitable (not perfectly competitive) direct insurance market, the expected return on shareholder investment could decrease with an increase in systematic risk and the incentive to fully diversify insurance activities may not be as high as that in a perfectly competitive insurance market.

2.3. Insurer default probability in case of a market shortfall

The analysis in the previous section illustrates that the Solvency Capital Requirement based on a Value at Risk measure places an upper limit on the risk-reward profile that shareholders are able to reach. In addition, to reach this maximum feasible risk-reward profile, the insurer should diversify investment and insurance risks. In this section, the theoretical analysis is concluded, considering the insurance default probability in case of a market shortfall. It is shown that if a large proportion of insurers wishes to diversify their liability risk, in order to raise their risk-reward profile, a very strong systemic effect is induced by regulation.

Let us introduce the return on net asset given a specific market return as:

\[
(\bar{\eta}_S|\bar{\eta}_m = \bar{r}_m) \sim \varphi(\bar{r}_I + \beta_S [\bar{r}_m - \bar{r}_I], \sigma(\bar{\epsilon}_S))
\]

Hence, the conditional default probability given a specific market return is:
\[
\text{Prob}(\bar{r}_S < 0 \mid \bar{r}_m = \bar{r}_m) = q(\bar{r}_m) = N \left[ -\frac{E(\bar{r}_S \mid \bar{r}_m = \bar{r}_m)}{\sigma(\bar{r}_S \mid \bar{r}_m = \bar{r}_m)} \right] = N \left[ -\frac{r_{\ell} + \beta_S [\bar{r}_m - r_{\ell}]}{\sigma(\bar{e}_S)} \right] = \]

\[
= N \left[ -\frac{r_{\ell} + \beta_F \left( \frac{4\alpha}{S_0} (\beta_A - \beta_L) \right) [\bar{r}_m - r_{\ell}]}{\sqrt{\sigma^2(\bar{e}_F) + \frac{4\alpha}{S_0} \left( \sigma^2(\bar{e}_A) + \sigma^2(\bar{e}_L) \right)}} \right] \quad [22.]
\]

We are interested in analyzing this conditional default probability specifically within a VaR-SCR constraint. It can be demonstrated that if the insurer holds net assets \(S_0\) exactly equal to the VaR-SCR \(S^*_{0}\) and, for simplicity, investment risk is perfectly diversified and asset-liability risk is immunized, the conditional default probability given a specific market return is\(^6\):

\[
q(\bar{r}_m) = N \left[ -N^{-1} (1 - p) \cdot \frac{r_{\ell} + \beta_F [\bar{r}_m - r_{\ell}]}{\sqrt{[r_{\ell} + \beta_F [E(\bar{r}_m) - r_{\ell}]]^2 - (N^{-1} (1 - p) \beta_F \sigma(\bar{r}_m))^2}} \right] \quad [23.]
\]

Clearly, this conditional default probability inversely depends on market return. The better the performance of the market, the less the default probability is and vice versa. More importantly, the function \(q(\bar{r}_m)\) depends on systematic risk \(\beta_F\). There are two limiting cases depicted in figure 1. In the first case, the insurer has no systematic risk, i.e. \(\beta_F=0\). Here, the default probability is equal to \(p\), irrespective of the market return, i.e.:

\[
q(\bar{r}_m) = N[-N^{-1} (1 - p)] = p
\]

The second limiting case is where the insurer is fully diversified, i.e. only systematic risk is assumed. As illustrated in the previous section, this situation occurs when the insurer wishes to reach a risk-reward profile higher than feasible and the diversifiable risk is eliminated in order to reach the maximum allowed risk-return profile.

\(^6\) Equation [23] is obtained with simple algebra considering that \(\sigma(\bar{e}_F)=0\) and \(\beta_L = \beta_A\) and substitute for \(S^*_{0}\) of equation [17] in equation [22].
Allowing \( \hat{r} \) to be the p-percentile of the market return distribution, for a fully diversified insurer, the insurer default probability is:

- equal to zero when the market return is greater than \( \hat{r}_m \);
- equal to 0.5 when the market return is equal to \( \hat{r}_m \); and
- equal to one when the market return is less than \( \hat{r}_m \).

The effects of these limiting situations are of utmost importance. On the one hand, when the market drop exceeds the level of confidence fixed by the regulator, all fully diversified insurance companies enter into bankruptcy simultaneously. This is a very extreme systemic effect induced by capital requirements. On the other hand, in a deep market scenario the less diversified insurance companies have more resilience as a whole, since the capital requirement is not only based on systematic risks but also diversifiable risks.

Furthermore, the strength of the VaR capital requirement – i.e. the rise in the confidence level \( 1-p \) – makes the insurers’ default wave less likely but consistently stronger. Thus, a stronger capital requirement reduces the maximum expected return on equity that could be reached by the insurers and this makes the capital requirement binding for a larger number of insurers, providing them with incentives for higher diversification.

Figure 2 depicts the insurer’s conditional default probability given a specific market return, i.e. \( q(\hat{r}_m) \) in equation [23.], as a function of the diversification degree for five different levels of market shortfall. The preceding two limiting cases correspond to a diversification degree equal to zero and one respectively. In the first limiting case, the insurer default probability is equal to 0.5% (i.e. one unit less than the confidence level), independently to market return. In the second limiting case, the insurer:
- defaults with certainty when the market return is lower than in the 0.5% worst case scenario (which corresponds to a one year market return of less than 0.5848 or a market rate of return less than -41.52%);
- has a fifty/fifty default probability when the one year market return is exactly equal to 0.5848; and
- has zero default probability when the market shortfall is less than -41.52%.

For the less diversified insurers (for example with a diversification degree lower than 40%), the default probability remains relatively low even in a very deep market shortfall (for instance in the case of the 0.01% worst case scenario which corresponds to a one year market shortfall of -64.38%, the default probability is around 10%). On the contrary, the more diversified insurers (for example with a DG between 80% and 95%) have significant default probability (i.e. more than 10%) also in “normal” market shortfalls (for example in the 1% worst case scenario which corresponds to a one year market rate of return equal to -36.53%).

FIGURE #2 HERE

3. Policy implications and conclusive remarks

The theoretical model depicted in the previous section suggests that Solvency II regulation based on a Value at Risk measure has some significant drawbacks. Moreover:
- it provides insurance companies with incentives towards the financialization of insurance business;
- it amplifies the systemic exposure of the insurance industry to market shortfall.

The first adverse side effect is a consequence of the incentive towards diversification for insurance companies whose optimal risk-reward profile is higher than feasible. The complete diversification of insurance risks may be obtained in various ways. These include:
- growth (intra-LOB growth, inter-LOB growth or geographical diversification);
- a strategic refocusing of the business in the activities which implies a limited assumption of diversifiable insurance risks and allows for higher exposure to market risks (for example life business vs. non-life business);
- a shift towards insurance activities which implies an easier diversification of the risk insured (retail markets with highly homogeneous and insurable risks vs. corporate markets with less homogenous and insurable risks, such as cat risks);
- the revision of contractual clauses in insurance contracts in order to avoid or limit the insurance exposure to diversifiable insurance risks (for example, the study or promotion of insurance policies that transfer only nominal mortality or longevity risk to insurers).

The second adverse side effect is a consequence of the incentive to hold only systematic risks that, in case of a deep market shortfall (exceeding the confidence level required by regulators), lead all the diversified insurers – who are typically also the biggest - to bankruptcy.

Even if this model is based on simplified and somewhat unrealistic assumptions, the removal of the main hypothesis (normal distribution of risks and a single-period insurance business model) and the introduction of more realistic assumptions (high skewed insurance risk distribution or long-term insurance contracts and investments) strengthen rather than weaken the results outlined. More precisely, in the real world the scenarios exceeding a pre-specified level of confidence are ever more frequent than those predicted by the level of confidence based on either normal or other frequently used distributions. Furthermore, with negative skewed distributions the scenario exceeding the pre-specified level of confidence could be more seriously worsened than normal distribution predicts. With reference to the time horizon, in our single-period theoretical model insurers become bankrupted only when
asset cash inflows are less than claim cash outflows. In real markets, Solvency II measures the financial strength of the insurers over a one-year horizon which is based on fair value rather than cash flows. Therefore, an insurance company is bankrupted when the fair value of assets become less than the fulfilment value of liability, not considering the fact that the fair value of assets and the fulfilment value of liabilities also depend on specific markets conditions (i.e. current structure of interest rates, equity prices, credit spreads), even when unrelated to the fulfilment of contractual obligations in the long run.

Previous studies (cf. Baluch et al., 2011, Geneva Association, 2010) have concluded that the systemic risk in the insurance sector is only marginally relevant. However, the fact that the insurance sector, especially the European insurance market, is now and was in the past less exposed to systemic risk, does not imply that it can ever happen in the future, if the new regulatory system brings about this exposure.

Clearly, it is difficult to establish whether the severe prediction of this theoretical model would happen. However:
- the generalized banking crisis could be easily explained by this model (the systematic exposure to credit and market risks has led the “too big-to-fail”-diversified and levered financial institutions to a deep financial crisis when a highly negative “market return” has been realized);
- the fact that the discussion on Solvency II regulation has not paid attention to the differences between diversifiable and systematic risk is not encouraging;
- in the presence of such deep drawbacks, which could lead to the crisis of a vast part of the European insurance sector in a near or distant future, a general precautionary principle needs to be applied.
Finally, how can the Solvency II Framework be adjusted in order to minimize or avoid the consequences foreseen by this model?

The most intuitive solutions do not seem to be easily viable or effective. Let us examine these findings a little more closely.

First, the model has shown a strong trade-off between competitive insurance markets and financial stability. If insurance activities (i.e. the effective assumption of underwriting insurance risks) are profitable, the insurer’s incentive to become fully diversified is compensated by the incentive to exploit business opportunities. However, requesting regulators to introduce some anti-competitive measures in order to avoid a potential systemic effect does not seem to be easily viable, at least before such an effect happens.

Second, the theoretical model suggests a different treatment between systematic and diversifiable risks. More specifically, systematic risk capital requirements should be strengthened and in turn, diversifiable risk capital requirements should be weakened. For instance, the Solvency II level of confidence on systematic risk could be raised to 99.9% while the level of confidence on diversifiable risk could be decreased to 95%. Even if the Solvency II framework does not differentiate between systematic and diversifiable risk, this solution seems viable in practice. The modular Solvency II standard formula requires the calibration of the parameters of each sub-module in order to obtain a proxy for the Value at Risk over a one-year time horizon with a 99.5% level of confidence. An analysis should be performed in order to detect the sub-modules that imply the assumption of significant systematic risks. Some market risk sub-modules – such as equity, interest rate and spread -, credit risk and some life underwriting risk sub-modules (for example lapse) may be considered as the first candidates.
Although this solution seems to be viable, with only minor technical modifications in the Solvency II framework, it does not appear to be effective in practice. On the one hand, the strengthening of systematic risk requirements reduces the probability of default from 0.5% to a lower level (say for example 0.1%). On the other hand, it lowers the maximum risk-reward profile reachable by the insurers and a larger number of insurers is expected to have the incentive to fully diversify their business. Therefore, in the case of a market shortfall that exceeds the new confidence level, the systemic financial crisis of the insurance sector is not deterred but rather encouraged. This solution reduces the probability of a systemic crisis but it does not mitigate the crisis of the insurance sector in the case of a deep market shortfall.

Whereas a practical differentiation between the Solvency Capital Requirement on systematic risk and diversifiable risk does not appear to be practically effective, the analyses of the systematic nature of insurance company risks should be performed carefully. This is in view of the fact that it is important to avoid a calibration that has a favourable treatment of systematic risk. One example of such a misconception is the calibration of the equity submodule in the QIS5 specification. Hence, the basic capital requirement on equity is reduced to 39% (cf. European Commission, 2010), even if the parameter estimated and recommended by the EIOPA, which is 42%, is based on hypothesis (normal distribution and long-run-historical estimates) that already undermine future equity risk (cf. Ceiops, 2010).

A simple, viable and possibly effective solution to the theoretical issues posed by this paper, is to set out that a significant part of the Solvency Capital Requirement be referred to diversifiable (insurance) risks. If the quota of diversifiable (insurance) risk drops below a predefined threshold, further diversification benefits must not be considered in the SCR calculation. The technical solutions in order to introduce such limits are not addressed in this paper. However, some comments should be made on the positive consequences of this
proposal. To this end, it could be useful to illustrate the effects of this measure in figure 3 which mirrors figure 2 apart from the fact that when a diversification degree of 60% is overwhelming, no further diversification effects are admitted to the SCR calculation. This avoids further assumption of systematic risk and as diversification continues, the insurance companies are less (not more) exposed to market shortfalls, since diversification reduces total risks but has no effects on the reduction of the SCR. Consequently, all the previously illustrated side effects are null and void. Moreover:

- this measure reduces the incentive towards the financialization of insurance activities;
- the bigger-better diversified insurance companies are required to be less risky than the smaller-worse diversified insurers; thus, it reduces the too-big-to-fail issue and the systemic exposure of the insurance industry to market shortfalls, contributing to the effective achievement of the proportionality principle while possibly permitting a higher credit rating for the bigger-better diversified insurer or better still, a reduction in the overall confidence level required by the regulators (for example from 99.5% to 99% or less).

FIGURE #3 HERE

Solvency II introduces a revolutionary risk management framework for the European insurance industry. Its basic more risk - more capital principle is simple and unanimously shareable. However, the technical specification of the principle is far from simple and requires a careful risk measurement. Here, the metric used for measurement is the most important issue. This paper has shown that the metric used by regulators, the Value at Risk, is not a balanced solution between effectiveness and simplicity, but is simply wrong and could lead to significant adverse side effects, ultimately resulting in a generalized European insurance industry crisis in the case of a hard market shortfall. Therefore, some adjustments
to the Solvency II framework are necessary before the new regulatory system is enforced. Invariably, academic predictions and suggestions regarding the pitfalls in risk measurement and risk management systems are taken on board and endorsed only after the foreseen side effects have occurred, as in the case of Cassandra in the Greek myth. Let us hope that is not the case.

4. References


Figure 1

**Insurer default probability as a function of market return: the two limiting cases**

This figure depicts the default probability of a fully diversified insurer (black line) and of an insurer without systematic risk (grey line) as a function of market return. The insurer operates with a net asset value equal to the Value at Risk requirement with a confidence level equal to 1-p. The market return density distribution is depicted in the upper part of the figure.
Figure 2

Insurer conditional default probability given the market return as a function of the diversification degree

This figure shows the insurer default probability (left-y-axis scale) as a function of the diversification degree (cf. Eq. [23.]) for five different market shortfalls (worst case scenarios). The parameters of the model are: \( r_f = 1.04; \) \[ E(\tilde{r}_m) - r_f = 6\%; \] \( \sigma(\tilde{r}_m) = 20\%; \) \( E(L) = 100; \) \( \beta_L = -0.1; \) \( \sigma(\tilde{e}_L) = 0; \) \( \beta_F = 0; \) \( F_0 = S_0 = 20; \) \( \sigma(\tilde{e}_F) = 0. \) The Solvency Capital Requirement of the insurance company is the VaR with a 99.5% confidence level and is computed by using eq. [17.]. \( \sigma(\tilde{e}_L) \) and \( \beta_F \) are variable across the x-axis in order to obtain a SCR exactly equal to the existing net asset value. For example, on the left hand side of the graph (degree of diversification = 0) \( \sigma(\tilde{e}_L) = 8.35\% \) and \( \beta_F = 0, \) while on the right hand side (full diversification) \( \sigma(\tilde{e}_L) = 0\% \) and \( \beta_F = 2.28480. \) The expected rate of return on equity is also depicted as a function of the diversification degree (right-y-axis).
**Figure 3**

*Insurer conditional default probability given the market return as a function of the diversification degree: SCR calculation with limits in the recognition of the diversification effects*

This figure shows the insurer default probability (left-y-axis scale) as a function of the diversification degree (cf. Eq. [23.]) for five different market shortfalls (worst case scenarios). Each parameter of the model is the same as figure 2 apart from the Solvency Capital Requirement calculation. The Solvency Capital Requirement of the insurance company is the VaR with a 99.5% confidence level and is computed by using eq. [17.] up to a diversification degree equal to 60% (which corresponds to $\beta_r=1.29$ and $\sigma(\delta_1)=7.21\%$). For a higher diversification degree ($\sigma(\delta_1)<7.21\%$), the Solvency Capital Requirement is not lowered. Therefore the maximum systematic risk exposure compatible with a SCR=$S_0=20$ is constant and equal to $\beta_r=1.29$ even if diversifiable risk decreases.